

**Section 1: Angle Measures**

Convert each measure into degrees, minutes, and seconds. Round to the nearest second.

1.  $23.653^\circ$  #1  $23^\circ$   
 $.653 \times 60 = 39.18'$   
 $.18 \times 60 = 10.8''$   
 $23^\circ 39' 11''$

2.  $245.417^\circ$  #2  $245^\circ$   
 $.417 \times 60 = 25.02'$   
 $.02 \times 60 = 1.2''$   
 $245^\circ 25' 1''$

Convert from degrees, minutes, and seconds to decimal form. Round to the nearest thousandths.

3.  $25^\circ 12' 45''$  #3  $45/60 = .75$   
 $12.75/60 = .2125$   
 $25.2125$   
 $25.213^\circ$

4.  $96^\circ 29' 11''$  #4  $11/60 = .18\bar{3}$   
 $29.18\bar{3}/60 = .4863\bar{8}$   
 $96.4863\bar{8}$   
 $96.486^\circ$

Convert from degrees to radians.

5.  $250^\circ$  #5  $250 \left[ \frac{\pi}{180} \right] = \frac{25\pi}{18}$

6.  $125^\circ$  #6  $125 \left[ \frac{\pi}{180} \right] = \frac{25\pi}{36}$

Convert from radians to degrees

7.  $\frac{7}{3}\pi$  #7  $\frac{7}{3}\pi \left[ \frac{180}{\pi} \right] = 420^\circ$

8.  $\frac{4}{5}\pi$  #8  $\frac{4}{5}\pi \left[ \frac{180}{\pi} \right] = 144^\circ$

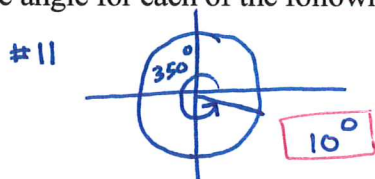
Find the standard position of each angle with coterminal side found  $[0^\circ, 360^\circ]$ , also identify which quadrant the angle falls in.

9.  $8427^\circ$  #9  $8427/360 = 23.408\bar{3}$   
 $8427 - 23[360] = 147^\circ$  Q II

10.  $2692^\circ$  #10  $2692/360 = 7.47$   
 $2692 - 7[360] = 172^\circ$  Q II

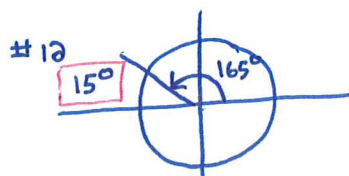
Find the reference angle for each of the following

11.  $350^\circ$



REFERENCE ANGLE ALWAYS MEASURED FROM X-AXIS

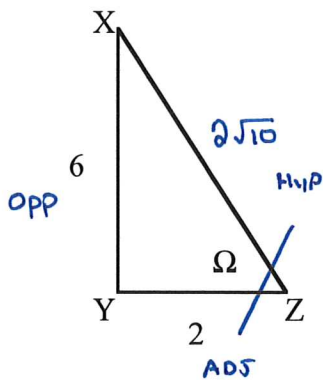
12.  $165^\circ$



## Section 2: Trig Functions

Find the values for the six trig functions.

13.



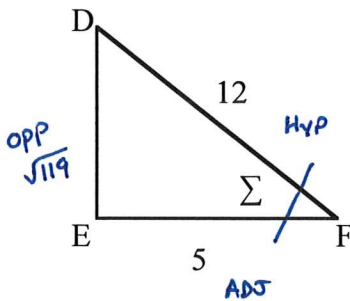
$$\begin{aligned} \text{HYP}^2 &= 6^2 + 2^2 \\ \text{HYP}^2 &= 36 + 4 \\ \text{HYP} &= \sqrt{40} \\ \text{HYP} &= 2\sqrt{10} \end{aligned}$$

$$\sin \Omega = \frac{6}{2\sqrt{10}} = \frac{3\sqrt{10}}{10} \quad \csc \Omega = \frac{\sqrt{10}}{3}$$

$$\cos \Omega = \frac{2}{2\sqrt{10}} = \frac{\sqrt{10}}{10} \quad \sec \Omega = \sqrt{10}$$

$$\tan \Omega = \frac{6}{2} = 3 \quad \cot \Omega = \frac{1}{3}$$

14.



$$\begin{aligned} 12^2 &= 5^2 + f^2 \\ 144 - 25 &= f^2 \\ 119 &= f^2 \\ \sqrt{119} &= f \end{aligned}$$

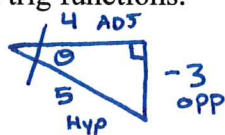
$$\sin \Sigma = \frac{\sqrt{119}}{12} \quad \csc \Sigma = \frac{12\sqrt{119}}{119}$$

$$\cos \Sigma = \frac{5}{12} \quad \sec \Sigma = \frac{12}{5}$$

$$\tan \Sigma = \frac{\sqrt{119}}{5} \quad \cot \Sigma = \frac{5\sqrt{119}}{119}$$

Find the values for the six trig functions.

15.  $(4, -3)$  #15



$$\sin \theta = -3/5 \quad \csc \theta = -5/3$$

$$\cos \theta = 4/5 \quad \sec \theta = 5/4$$

$$\tan \theta = -3/4 \quad \cot \theta = -4/3$$

16.  $(0, 7)$

#16

RECALL  $x = \cos \theta$   
 $y = \sin \theta$

$$\sin \theta = 7$$

$$\cos \theta = 0$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \text{UNDEFINED}$$

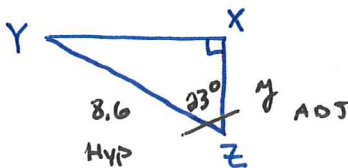
$$\csc \theta = 1/7$$

$$\sec \theta = \text{UNDEFINED}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta} = 0$$

Solve the following using right triangle trig.

17. In  $\triangle XYZ$ ,  $X = 90^\circ$  find  $y$  to the nearest tenth if  $Z = 23^\circ$  and  $x = 8.6$

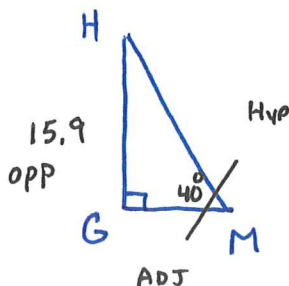


$$\cos 23^\circ = \frac{y}{8.6}$$

$$8.6 \cos 23^\circ = y$$

$$\underline{7.9 = y}$$

18. In  $\triangle HGM$ ,  $G = 90^\circ$  find  $h$  to the nearest tenth if  $M = 40^\circ$  and  $m = 15.9$

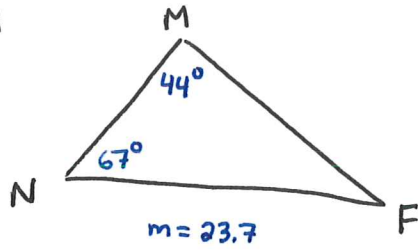


$$\tan 40^\circ = \frac{15.9}{h}$$

$$h = \frac{15.9}{\tan 40^\circ}$$

$$\underline{h = 18.9}$$

#19



$$m \angle F = 180 - 67 - 44$$

$$m \angle F = 69^\circ$$

$$\frac{n}{\sin 67^\circ} = \frac{23.7}{\sin 44^\circ}$$

$$n = \frac{23.7 \sin 67^\circ}{\sin 44^\circ}$$

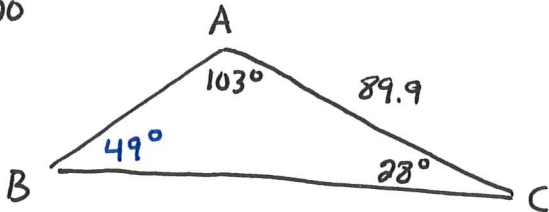
$$n = 31.4$$

$$\frac{f}{\sin 69^\circ} = \frac{23.7}{\sin 44^\circ}$$

$$f = \frac{23.7 \sin 69^\circ}{\sin 44^\circ}$$

$$f = 31.9$$

#20



$$m \angle B = 180^\circ - 103^\circ - 28^\circ$$

$$m \angle B = 49^\circ$$

$$\frac{c}{\sin 28^\circ} = \frac{89.9}{\sin 49^\circ}$$

$$c = \frac{89.9 \sin 28^\circ}{\sin 49^\circ}$$

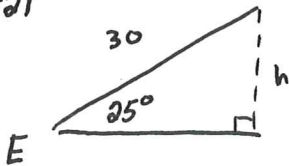
$$c = 55.9$$

$$\frac{a}{\sin 103^\circ} = \frac{89.9}{\sin 49^\circ}$$

$$a = \frac{89.9 \sin 103^\circ}{\sin 49^\circ}$$

$$a = 116.1$$

#21



$$e = 23.7$$

CALCULATE MINIMUM HEIGHT

$$\sin 25 = \frac{h}{30}$$

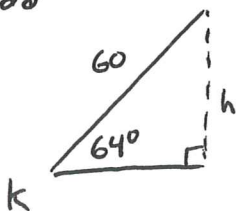
$$30 \sin 25 = h$$

$$12.7 = h$$

SINCE  $e = 23.7$  FALLS BETWEEN MINIMUM  $h = 12.7$  AND  $f = 30$   
THIS WOULD CONSTITUTE AMBIGUOUS CASE.

THEREFORE, TWO TRIANGLES POSSIBLE.

#22



$$k = 45.3$$

CALCULATE MINIMUM HEIGHT

$$\sin 64^\circ = \frac{h}{60}$$

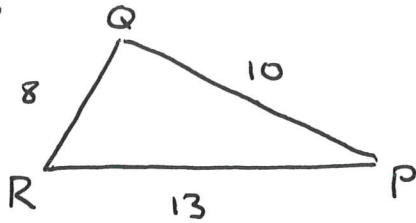
$$60 \sin 64^\circ = h$$

$$53.9 = h$$

SINCE  $k = 45.3$  FALLS SHORT OF MINIMUM  
 $h = 53.9$

NO TRIANGLE CAN EXIST.

#23



LAW OF SINES

$$\frac{\sin P}{8} = \frac{\sin 91.8^\circ}{13}$$

$$\sin P = \frac{8 \sin 91.8^\circ}{13}$$

$$P = \sin^{-1} \left[ \frac{8 \sin 91.8^\circ}{13} \right]$$

$$P = 38.0^\circ$$

LAW OF COSINES

$$q^2 = r^2 + p^2 - 2rp \cos Q$$

$$13^2 = 10^2 + 8^2 - 2(10)(8) \cos Q$$

$$\frac{13^2 - 10^2 - 8^2}{-2(10)(8)} = \cos Q$$

$$\cos^{-1} \left[ \frac{13^2 - 10^2 - 8^2}{-2(10)(8)} \right] = Q$$

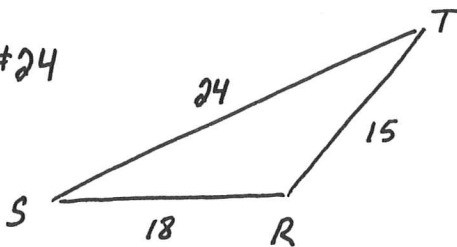
$$91.8^\circ \approx Q$$

$$m \angle Q = 91.8^\circ$$

$$m \angle P = 38.0^\circ$$

$$m \angle R = 50.2^\circ$$

#24



LAW OF SINES

$$\frac{\sin T}{18} = \frac{\sin 92.9^\circ}{24}$$

$$\sin T = \frac{18 \sin 92.9^\circ}{24}$$

$$T = \sin^{-1} \left[ \frac{18 \sin 92.9^\circ}{24} \right]$$

$$T = 48.5^\circ$$

LAW OF COSINES

$$r^2 = s^2 + t^2 - 2st \cos R$$

$$24^2 = 15^2 + 18^2 - 2(15)(18) \cos R$$

$$\frac{24^2 - 15^2 - 18^2}{-2(15)(18)} = \cos R$$

$$\cos^{-1} \left[ \frac{24^2 - 15^2 - 18^2}{-2(15)(18)} \right] = R$$

$$92.9^\circ \approx R$$

$$m \angle R = 92.9^\circ$$

$$m \angle T = 48.5^\circ$$

$$m \angle S = 38.6^\circ$$

### Section 3: Trig Laws

Using law of sines solve the triangle to the nearest tenth.

19. In  $\triangle NMF$ ,  $N = 67^\circ$ ,  $M = 44^\circ$ , and  $m = 23.7$

20. In  $\triangle ABC$ ,  $A = 103^\circ$ ,  $C = 28^\circ$ , and  $b = 89.9$

Determine the number of triangles possible.

21. In  $\triangle EFK$ ,  $E = 25^\circ$ ,  $f = 30$ , and  $e = 23.7$

22. In  $\triangle JKL$ ,  $K = 64^\circ$ ,  $j = 60$ , and  $k = 45.3$

Using law of cosines solve the triangle to the nearest tenth for all angles.

23. In  $\triangle PQR$ ,  $p = 8$ ,  $q = 13$ , and  $r = 10$

24. In  $\triangle RST$ ,  $r = 24$ ,  $s = 15$ , and  $t = 18$

### Section 4: Classifying Triangles

Consider the lengths of the sides for the proposed triangle. First determine if the triangle can exist, then classify the triangle as acute, right, or obtuse. If no triangle is possible then state it as such.

25.  $d = 5$ ,  $e = 7$ ,  $f = 13$

#25  $\left. \begin{array}{l} 5 + 7 = 12 \\ 7 - 5 = 2 \end{array} \right\} 13 \text{ DOES NOT FALL IN RANGE}$   
 $\therefore \text{NO TRIANGLE CAN BE FORMED}$   
NO TRIANGLE

26.  $a = 12$ ,  $b = 13$ ,  $c = 5$

#26  $\left. \begin{array}{l} 12 + 13 = 25 \\ 13 - 12 = 1 \end{array} \right\} 5 \text{ FALLS WITHIN RANGE}$   
 $\therefore \text{TRIANGLE EXISTS}$

27.  $x = 6$ ,  $y = 9$ ,  $z = 14$

$5^2 + 12^2 \quad 13^2$   
 $25 + 144 = 169 \Rightarrow \text{RIGHT TRIANGLE}$

28.  $q = 22$ ,  $r = 20$ ,  $t = 18$

#27  $\left. \begin{array}{l} 6 + 9 = 15 \\ 9 - 6 = 3 \end{array} \right\} 14 \text{ FALLS WITHIN RANGE}$   
 $\therefore \text{TRIANGLE EXISTS}$

$6^2 + 9^2 \quad 14^2$   
 $36 + 81 < 196 \Rightarrow \text{OBTUSE TRIANGLE}$

#28  $\left. \begin{array}{l} 20 + 18 = 38 \\ 20 - 18 = 2 \end{array} \right\} 22 \text{ FALLS WITHIN RANGE}$   
 $\therefore \text{TRIANGLE EXISTS}$

$18^2 + 20^2 \quad 22^2$   
 $324 + 400 > 484 \Rightarrow \text{ACUTE TRIANGLE}$

### Section 5: Areas of Triangles

Use the determinant of a matrix to find the area of a triangle.

29. Vertices are  $(2, -3)$ ,  $(7, -2)$ ,  $(-4, 9)$

30. Vertices are  $(6, -2)$ ,  $(11, 4)$ ,  $(1, -5)$

Use Heron's Formula to find the area of a triangle.

31.  $g = 11$ ,  $h = 4$ ,  $k = 7$

32.  $a = 9$ ,  $b = 13$ ,  $c = 6$

Use trig to find the area of the triangle.

33.  $A = 20^\circ$ ,  $a = 19$ ,  $C = 64^\circ$

34.  $x = 66$ ,  $y = 90$ ,  $Z = 58^\circ$

### Section 6: Finding Solutions by Inspection

This is a review of the last chapter. Be sure to check out zeros or restrictions along with proposed answers from argument on the number line to test for final solutions.

35.  $\sqrt{x-4} + 3 \leq 8$

36.  $\frac{3}{x-3} - \frac{5}{x+4} > \frac{2}{3}$

37.  $\frac{17}{x^2-25} < \frac{7}{x-5} + \frac{3}{x+5}$

#29

$$\begin{aligned} \text{AREA OF } \Delta &= \frac{1}{2} \det \begin{vmatrix} 2 & -3 & 1 \\ 7 & -2 & 1 \\ -4 & 9 & 1 \end{vmatrix} \\ &= \underline{33 \text{ UNITS}^2} \end{aligned}$$

#30

$$\begin{aligned} \text{AREA OF } \Delta &= \frac{1}{2} \det \begin{vmatrix} 6 & -2 & 1 \\ 11 & 4 & 1 \\ 1 & -5 & 1 \end{vmatrix} \\ &= \underline{7.5 \text{ UNITS}^2} \end{aligned}$$

#31 HERON'S FORMULA

$$\begin{aligned} s &= \frac{11+4+7}{2} \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{AREA} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{11(11-11)(11-4)(11-7)} \\ &= 0 \end{aligned}$$

THOSE SIDES CANNOT FORM A TRIANGLE, HENCE NO AREA

$$\left. \begin{array}{l} 11+4=15 \\ 11-4=7 \end{array} \right\} \text{THIRD SIDE MUST BE } \in (7, 15)$$

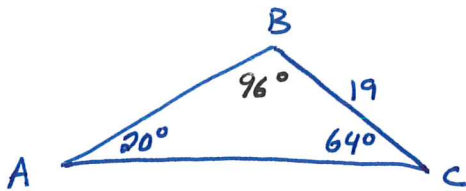
IN THIS CASE IT EQUALS 7  
SO NO TRIANGLE POSSIBLE

#32 HERON'S FORMULA

$$\begin{aligned} s &= \frac{9+13+6}{2} \\ &= 14 \end{aligned}$$

$$\begin{aligned} \text{AREA} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{14(14-9)(14-13)(14-6)} \\ &= \underline{23.7 \text{ UNITS}^2} \end{aligned}$$

#33 AREA USING LAW OF SINES



$$\text{AREA} = \frac{1}{2} b h$$

$$\text{AREA} = \frac{1}{2} \left[ \frac{19 \sin 96^\circ}{\sin 20^\circ} \right] \left[ 19 \sin 64^\circ \right]$$

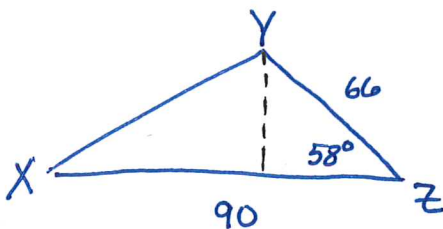
$$\frac{19}{\sin 20^\circ} = \frac{b}{\sin 96^\circ} \quad \left. \vphantom{\frac{19}{\sin 20^\circ}} \right\} \text{FIND BASE}$$
$$\frac{19 \sin 96^\circ}{\sin 20^\circ} = b$$

$$\underline{\text{AREA} = 471.7 \text{ UNITS}^2}$$

$$\sin 64^\circ = \frac{h}{19}$$
$$19 \sin 64^\circ = h$$

FIND HEIGHT

#34



$$\text{AREA} = \frac{1}{2} b h$$

$$= \frac{1}{2} [90] [66 \sin 58^\circ]$$

$$= \underline{2518.7 \text{ UNITS}^2}$$



$$\sin 58^\circ = \frac{h}{66}$$
$$66 \sin 58^\circ = h$$

#35

$$\sqrt{x-4} + 3 \leq 8$$

a. RESTRICTED VALUE  $x \geq 4$  IN ORDER TO HAVE REAL SOLUTIONS UNDER THE RADICAL

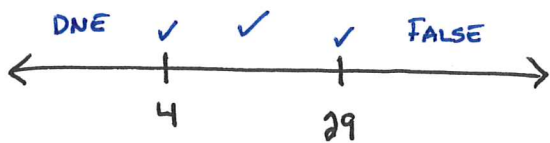
b.  $\sqrt{x-4} \leq 8-3$

$$\sqrt{x-4} \leq 5$$

$$x-4 \leq 25$$

$$x \leq 29$$

c. CHECK PROPOSED SOLUTIONS ON NUMBER LINE AGAINST ORIGINAL STATEMENT



$$D: [4, 29]$$

#36

$$\frac{3}{x-3} - \frac{5}{x+4} > \frac{2}{3}$$

a. RESTRICTED VALUES  $x \neq \{-4, 3\}$  TO AVOID DIVIDING BY ZERO

b.  $\left[ \frac{3}{x-3} - \frac{5}{x+4} > \frac{2}{3} \right] (x-3)(x+4) 3$  : MULTIPLY BY COMMON DENOMINATOR

$$3(x+4) \cdot 3 - 5(x-3) \cdot 3 > 2(x-3)(x+4)$$

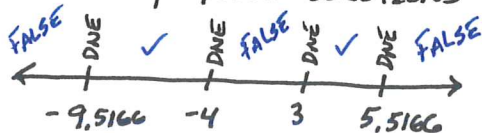
$$9x + 36 - 15x + 45 > 2(x^2 + x - 12)$$

$$-6x + 81 > 2x^2 + 2x - 24$$

$$0 > 2x^2 + 8x - 105$$

$$x = \{-9.5166, 5.5166\}$$

c. CHECK PROPOSED SOLUTIONS ON NUMBER LINE AGAINST ORIGINAL STATEMENT



\* DNE BECAUSE STRICT INEQUALITY

$$D: (-9.5166, -4) \cup (3, 5.5166)$$

#37 
$$\frac{17}{x^2-25} < \frac{7}{x-5} + \frac{3}{x+5}$$

a. RESTRICTED VALUES  $x \neq \{-5, 5\}$  TO AVOID DIVIDING BY ZERO

b. 
$$\left[ \frac{17}{x^2-25} < \frac{7}{x-5} + \frac{3}{x+5} \right] (x-5)(x+5)$$

$$17 < 7(x+5) + 3(x-5)$$

$$17 < 7x + 35 + 3x - 15$$

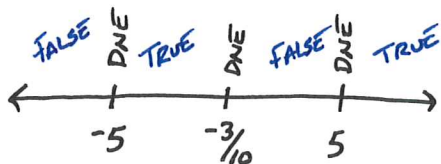
$$17 < 10x + 20$$

$$17 - 20 < 10x$$

$$-3 < 10x$$

$$-3/10 < x$$

c. CHECK PROPOSED SOLUTIONS ON NUMBER LINE



\*  $x = -3/10$  DNE BECAUSE STRICT INEQUALITY

$$M_1 = \frac{17}{x^2-25} - \frac{7}{x-5} - \frac{3}{x+5}$$

\* CHECKING FOR  $M_1 < 0$  (NEGATIVE)

$$D: (-5, -3/10) \cup (5, \infty)$$