

Section 1: Determining Consistent, Inconsistent and Dependent, Independent Lines

Consider the system of equations and identify the relationship as indicated above.

$$\begin{array}{l}
 2x - 3y = 11 \quad m = \frac{2}{3} \\
 1. \quad y = \frac{4}{5}x + 2 \quad m = \frac{4}{5}
 \end{array}
 \left. \vphantom{\begin{array}{l} 2x - 3y = 11 \\ y = \frac{4}{5}x + 2 \end{array}} \right\}
 \begin{array}{l}
 \underline{\text{LINES MUST CROSS}} \\
 \text{CONSISTENT} \\
 \underline{\text{DIFFERENT SLOPES}} \\
 \text{INDEPENDENT}
 \end{array}$$

1. CONSISTENT / INDEPENDENT

$$\begin{array}{l}
 x - 5y = 15 \quad m = \frac{1}{5} \\
 2. \quad y = \frac{1}{5}x + 2 \quad m = \frac{1}{5}
 \end{array}
 \left. \vphantom{\begin{array}{l} x - 5y = 15 \\ y = \frac{1}{5}x + 2 \end{array}} \right\}
 \begin{array}{l}
 \underline{\text{LINES PARALLEL}} \\
 \text{INCONSISTENT} \\
 \underline{\text{SAME SLOPE}} \\
 \text{DEPENDENT}
 \end{array}$$

2. INCONSISTENT / DEPENDENT

$$\begin{array}{l}
 6x + 7y = 14 \quad m = -\frac{6}{7} \\
 3. \quad y = -\frac{6}{7}x + 2 \quad m = -\frac{6}{7}
 \end{array}
 \left. \vphantom{\begin{array}{l} 6x + 7y = 14 \\ y = -\frac{6}{7}x + 2 \end{array}} \right\}
 \begin{array}{l}
 \underline{\text{LINES COINCIDE}} \\
 \text{CONSISTENT} \\
 \underline{\text{SAME SLOPE}} \\
 \text{DEPENDENT}
 \end{array}$$

3. CONSISTENT / DEPENDENT

Section 2: Solving Systems with Augmented Matrices

Write an **augmented matrix** for the system of equations then **solve** using the **reduced row echelon form** option in the calculator. **Rational solutions** should be represented as **proper fractions** or **mixed numbers** when appropriate.

$$\begin{array}{l}
 4x + y - 6z = -1 \\
 4. \quad 8x + 3y - 3z = -2 \\
 12x - 5y + 9z = 25
 \end{array}$$

ENTER 3 x 4 MATRIX
THEN USE RREF OPTION

$$4. \quad \begin{bmatrix} 4 & 1 & -6 & -1 \\ 8 & 3 & -3 & -2 \\ 12 & -5 & 9 & 25 \end{bmatrix}$$

Solution: $(\frac{107}{132}, -\frac{26}{11}, \frac{28}{99})$

$$\begin{array}{l}
 2x - 8y + 10z = -11 \\
 5. \quad 10x + 12y - 15z = 48 \\
 4x - 4y + 5z = 2
 \end{array}$$

USING RREF

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1.25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \left. \vphantom{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1.25 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \right\} \text{NO SOLUTION}$$

$$5. \quad \begin{bmatrix} 2 & -8 & 10 & -11 \\ 10 & 12 & -15 & 48 \\ 4 & -4 & 5 & 2 \end{bmatrix}$$

Solution: NO SOLUTION

Section 2: Operations with Matrices

Given the Following Matrices, complete the indicated operations.

$$A = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 9 & -3 \\ -1 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 5 & 8 & -6 \\ -2 & -4 & 10 \end{bmatrix} \quad D = \begin{bmatrix} -3 & -4 & -4 & 2 \\ 6 & -7 & 11 & 3 \\ 5 & 6 & 9 & 14 \end{bmatrix}$$

$$E = \begin{bmatrix} -2 & 3 & 1 & 1 \\ 5 & 6 & 7 & -11 \\ -5 & 8 & -7 & -3 \end{bmatrix} \quad F = \begin{bmatrix} -3 & -2 & 4 & 2 \\ 3 & 3 & 6 & -2 \\ -1 & -6 & 7 & 13 \end{bmatrix} \quad G = \begin{bmatrix} -6 & 5 & -3 \\ -2 & -1 & 4 \end{bmatrix}$$

6. Find $3E - 4F$

TYPE INTO CALCULATOR

$$3[E] - 4[F]$$

6.

$$\begin{bmatrix} 6 & 17 & -13 & -5 \\ 3 & 6 & -3 & -25 \\ -11 & 48 & -49 & -61 \end{bmatrix}$$

7. Find Matrix X if $4X + 3C = 2G$

SOLVE FIRST $4[X] = 2[G] - 3[C]$
 $X = \frac{1}{4}(2[G] - 3[C])$

THEN ENTER INTO CALCULATOR

7.

$$\begin{bmatrix} -6\frac{3}{4} & -3\frac{1}{2} & 3 \\ \frac{1}{2} & 2\frac{1}{2} & -5\frac{1}{2} \end{bmatrix}$$

8. Find Matrix K if $CE + 3G[F + D] = BK$

SOLVE FIRST $[B]^{-1}([C][E] + 3[G]([F] + [D]))$

THEN ENTER INTO CALCULATOR

8.

$$\begin{bmatrix} 31\frac{6}{11} & 16\frac{6}{11} & 29\frac{2}{3} & -16\frac{5}{33} \\ 5\frac{7}{11} & 28\frac{7}{11} & 17\frac{7}{3} & 73\frac{7}{33} \end{bmatrix}$$

9. Find $|A| + 5|B|$

(Determinant of A plus five times determinant of B)

$$\det([A]) + 5 \times \det([B])$$

9. 206

10. Solve the system of equations generated from these determinants.

$$\begin{vmatrix} 2x+3 & 4 \\ 4y-3 & -5 \end{vmatrix} = 8 \qquad \begin{vmatrix} 5x+3 & -2 \\ 2y-5 & -3 \end{vmatrix} = -11$$

$$\begin{aligned} -5(2x+3) - 4(4y-3) &= 8 & -3(5x+3) - 2(2y-5) &= -11 \\ -10x - 15 - 16y + 12 &= 8 & -15x - 9 + 4y - 10 &= -11 \\ -10x - 16y &= 11 & -15x + 4y &= 8 \end{aligned}$$

SETUP SYSTEM & SOLVE

10. $\left(\frac{-43}{70}, \frac{-17}{56} \right)$

11. Solve the system of equations generated from these determinants.

$$\begin{vmatrix} 6 - \frac{1}{2}x & 5 \\ y+2 & 3 \end{vmatrix} = 3\frac{1}{4} \qquad \begin{vmatrix} 2 - \frac{3}{4}x & -3 \\ \frac{1}{3}y+2 & 5 \end{vmatrix} = -2\frac{1}{3}$$

$$\begin{aligned} -6x - 20y &= -19 \\ -45x + 12y &= -220 \end{aligned}$$

$$\begin{aligned} 3(6 - \frac{1}{2}x) - 5(y+2) &= 3\frac{1}{4} & 5(2 - \frac{3}{4}x) - 3(\frac{1}{3}y+2) &= -2\frac{1}{3} \\ 18 - \frac{3}{2}x - 5y - 10 &= 3\frac{1}{4} & 10 - \frac{15}{4}x + y + 6 &= -2\frac{1}{3} \\ 4[18 - \frac{3}{2}x - 5y - 10 = 3\frac{1}{4}] & & 12[10 - \frac{15}{4}x + y + 6 = -2\frac{1}{3}] & \\ 72 - 6x - 20y - 40 &= 13 & 120 - 45x + 12y + 72 &= -28 \end{aligned}$$

11. $\left(4\frac{185}{243}, -\frac{155}{324} \right)$

12. Find the area of the triangle with vertices at (-2, 5), (3, 7), (11, 0)

$$\frac{1}{2} \begin{vmatrix} -2 & 5 & 1 \\ 3 & 7 & 1 \\ 11 & 0 & 1 \end{vmatrix} = \frac{1}{2} |-51| = 25\frac{1}{2}$$

12. $25\frac{1}{2} \text{ UNITS}^2$

13. Find the area of the triangle with vertices at (7, 3), (1, -2), (4, 5)

$$\frac{1}{2} \begin{vmatrix} 7 & 3 & 1 \\ 1 & -2 & 1 \\ 4 & 5 & 1 \end{vmatrix} = \frac{1}{2} |-27| = 13\frac{1}{2}$$

13. $13\frac{1}{2} \text{ UNITS}^2$

14. Solve the matrix Equation

$$\begin{bmatrix} -2 & 3 & 5 \\ 7 & -2 & 1 \\ 4 & 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 3 & 5 \\ 7 & -2 & 1 \\ 4 & 1 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 8 \\ -1 \\ 4 \end{bmatrix}$$

14. $\left(1\frac{11}{15}, 5\frac{14}{15}, -1\frac{4}{15} \right)$

15. Solve the matrix Equation

$$\begin{bmatrix} -1 & 3 & 4 \\ 10 & -6 & 1 \\ 4 & 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} - 3 \begin{bmatrix} 5 & 3 & 5 \\ 4 & -2 & 9 \\ 9 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 22 \\ -6 \\ 3 \end{bmatrix}$$

$$\left(\begin{bmatrix} -1 & 3 & 4 \\ 10 & -6 & 1 \\ 4 & 6 & 2 \end{bmatrix} - 3 \begin{bmatrix} 5 & 3 & 5 \\ 4 & -2 & 9 \\ 9 & 1 & 4 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 22 \\ -6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -16 & -6 & -11 \\ -2 & 0 & -26 \\ -23 & 3 & -10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 22 \\ -6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -16 & -6 & -11 \\ -2 & 0 & -26 \\ -23 & 3 & -10 \end{bmatrix}^{-1} \begin{bmatrix} 22 \\ -6 \\ 3 \end{bmatrix}$$

15. $\left(\frac{-457}{775}, -2\frac{1396}{2325}, 2\frac{14}{775} \right)$