

Test On Matrices (Pre Calculus) In-class

Name Key

Section 1: Determining Consistent, Inconsistent and Dependent, Independent Lines

Consider the system of equations and identify the relationship as indicated above.

1. $3x - 4y = 20$ $m = \frac{3}{4}$ } LINES COINCIDE
 $y = \frac{3}{4}x - 5$ $m = \frac{3}{4}$ } CONSISTENT
SAME SLOPE
DEPENDENT

1. CONSISTENT / DEPENDENT

2. $2x - 5y = 11$ } $m = \frac{2}{5}$ } LINES MUST CROSS
 $y = \frac{3}{4}x + 2$ } $m = \frac{3}{4}$ } CONSISTENT
DIFFERENT SLOPE
INDEPENDENT

2. CONSISTENT / INDEPENDENT

3. $x + 3y = 15$ } $m = -\frac{1}{3}$ } LINE PARALLEL
 $y = \frac{-1}{3}x + 2$ } $m = -\frac{1}{3}$ } INCONSISTENT
SAME SLOPE
DEPENDENT

3. INCONSISTENT / DEPENDENT

Section 2: Solving Systems with Augmented Matrices

Write an **augmented matrix** for the system of equations then **solve** using the **reduced row echelon form** option in the calculator. **Rational solutions** should be represented as **proper fractions** or **mixed numbers** when appropriate.

4. $6x - y + 2z = 6$
 $9x + 4y - 4z = 0$
 $12x - 7y + 8z = 19$

ENTER 3 x 4 MATRIX
 THEN USE RREF OPTION

4.
$$\begin{bmatrix} 6 & -1 & 2 & 6 \\ 9 & 4 & -4 & 0 \\ 12 & -7 & 8 & 19 \end{bmatrix}$$

Solution: $(\frac{2}{3}, -1, \frac{1}{2})$

5. $4x - 5y + 3z = -9$
 $6x + 20y - 2z = 28$
 $8x - 10y + 9z = -27$

5.
$$\begin{bmatrix} 4 & -5 & 3 & -9 \\ 6 & 20 & -2 & 28 \\ 8 & -10 & 9 & -27 \end{bmatrix}$$

Solution: $(1, \frac{4}{5}, -3)$

Section 2: Operations with Matrices

Given the Following Matrices, complete the indicated operations.

$$A = \begin{bmatrix} 2 & -5 \\ 4 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 6 & -8 \\ -2 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 8 & -4 \\ 2 & -4 & 10 \end{bmatrix} \quad D = \begin{bmatrix} -2 & -4 & -4 & 2 \\ 6 & -8 & 12 & 4 \\ 4 & 6 & 8 & 14 \end{bmatrix}$$

$$E = \begin{bmatrix} -5 & 2 & -4 & 1 \\ 3 & 6 & 7 & -13 \\ -5 & 8 & -7 & -2 \end{bmatrix} \quad F = \begin{bmatrix} -3 & -1 & 5 & 7 \\ 5 & 3 & 11 & -3 \\ -1 & -9 & 7 & 15 \end{bmatrix} \quad G = \begin{bmatrix} -7 & 3 & -5 \\ -3 & -9 & 1 \end{bmatrix}$$

6. Find $4E - 5F + 2D$

$$4 \times [E] - 5 \times [F] + 2 \times [D]$$

6.

$$\begin{bmatrix} -9 & 5 & -49 & -27 \\ -1 & -7 & -3 & -29 \\ -7 & 89 & -47 & -55 \end{bmatrix}$$

7. Find **Matrix Y** if $2X + 3C = 4G$

$$\text{Solve: } Y = \frac{1}{2} \times (4 \times [G] - 3 \times [C])$$

7.

$$\begin{bmatrix} -23 & -6 & -4 \\ -9 & -12 & -13 \end{bmatrix}$$

8. Find **Matrix L** if $GD - C[2F + 4D] = BL$

$$[B]^{-1} \times ([G] \times [D] - [C] \times (2 \times [F] + 4 \times [D])) = L$$

8.

$$\begin{bmatrix} -80 & 119 & -414 & -862 \\ -45 & 50 & -271 & -645 \end{bmatrix}$$

9. Find $3|A| - 5|B|$

(Three times determinant of A minus five times determinant of B)

9. 38

10. Solve the system of equations generated from these determinants. (Integer Coordinates)

$$\begin{vmatrix} 3x-2 & 3 \\ 5y+7 & -2 \end{vmatrix} = 25 \qquad \begin{vmatrix} 4x-7 & -2 \\ y+3 & 3 \end{vmatrix} = 13 \qquad \begin{bmatrix} -6 & -15 & 42 \\ 12 & 2 & 28 \end{bmatrix}$$

$$\begin{aligned} -2(3x-2) - 3(5y+7) &= 25 & 3(4x-7) + 2(y+3) &= 13 \\ -6x+4 - 15y-21 &= 25 & 12x-21 + 2y+6 &= 13 \\ -6x-15y &= 42 & 12x+2y &= 28 \end{aligned}$$

10. (3, -4)

11. Solve the system of equations generated from these determinants. (Integer Coordinates)

$$\begin{vmatrix} 4 - \frac{1}{3}x & -4 \\ 2y+3 & \frac{1}{3} \end{vmatrix} = 4\frac{1}{3} \qquad \begin{vmatrix} \frac{1}{2}x-4 & -3 \\ \frac{1}{3}y+5 & 4 \end{vmatrix} = 16 \qquad \begin{bmatrix} -\frac{1}{9} & 8 & -8 \\ 2 & 1 & 17 \end{bmatrix}$$

$$\begin{aligned} \frac{1}{3}(4 - \frac{1}{3}x) + 4(2y+3) &= 4\frac{1}{3} & 4(\frac{1}{2}x-4) + 3(\frac{1}{3}y+5) &= 16 \\ \frac{4}{3} - \frac{1}{9}x + 8y + 12 &= 4\frac{1}{3} & 2x-16 + y+15 &= 16 \\ -\frac{1}{9}x + 8y &= -8 & 2x+y &= 17 \end{aligned}$$

11. (9, -1)

12. Find the area of the triangle with vertices at (-3, 9), (8, 2), (5, 3)

$$\boxed{\frac{1}{2} \text{DET}([A])} \qquad [A] = \begin{bmatrix} -3 & 9 & 1 \\ 8 & 2 & 1 \\ 5 & 3 & 1 \end{bmatrix}$$

12. 5 UNITS²

13. Find the area of the triangle with vertices at (11, 5), (9, -2), (-2, 3)

$$\frac{1}{2} \text{DET}([A]) \qquad [A] = \begin{bmatrix} 11 & 5 & 1 \\ 9 & -2 & 1 \\ -2 & 3 & 1 \end{bmatrix}$$

13. 43½ UNITS²

14. Solve the matrix Equation $\begin{bmatrix} -4 & 1 & 3 \\ 2 & -3 & 5 \\ 5 & 6 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -17 \\ 29 \\ 3 \end{bmatrix}$

(Integer Solutions)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [A]^{-1}[B]$$

14. (5, -3, 2)

15. Solve the matrix Equation $2 \begin{bmatrix} 3 & -2 & 3 \\ 5 & -3 & 7 \\ 4 & 5 & 9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} - 3 \begin{bmatrix} 8 & -2 & 4 \\ 6 & -8 & 7 \\ 3 & 11 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -34 \\ 28 \\ -189 \end{bmatrix}$

(Integer Solutions)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = (2 \times [A] - 3 \times [B])^{-1} \times [C]$$

15. (4, 1, -6)

Section 4: Solving more Matrix Equations

#16 Solve the Matrix Equation

$$4 \cdot \begin{bmatrix} 3 & 5 & -1 \\ -2 & 9 & 7 \\ 8 & 3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} - 7 \cdot \begin{bmatrix} 1 & 1 & -2 \\ 3 & 1 & 5 \\ 4 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 3 & 1 & 2 \\ 5 & 5 & -3 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 128 \\ 202 \\ -28 \end{bmatrix}$$

$[A]$
 $[B]$
 $[C]$
 $[D]$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left(4[A] - 7[B] + [C] \right)^{-1} \times [D]$$

16. $(-2, 6, 5)$

#17 Solve the equation made by the determinant.

$$\begin{vmatrix} 4x-3 & x-7 \\ x+1 & x+2 \end{vmatrix} = 21$$

$$(4x-3)(x+2) - (x+1)(x-7) = 21$$

$$4x^2 + 5x - 6 - x^2 + 6x + 7 =$$

$$3x^2 + 11x + 1 = 21$$

$$3x^2 + 11x - 20 = 0$$

USE QUADRATIC FORMULA

$$x = \left\{ -5, 1\frac{1}{3} \right\}$$

17. $x = \left\{ -5, 1\frac{1}{3} \right\}$

#18 Solve the system generated by the determinants.

$$\begin{vmatrix} 3x+1 & 4y+2 \\ 11 & -2 \end{vmatrix} = -56 \quad \& \quad \begin{vmatrix} 3y-5 & 2 \\ x+4 & -3 \end{vmatrix} = 2$$

$$-2(3x+1) - 11(4y+2) = -56$$

$$-6x - 2 - 44y - 22 = -56$$

$$-6x - 44y = -32$$

USE AUGMENTED MATRIX

$$\begin{bmatrix} -6 & -44 & -32 \\ -2 & -9 & -5 \end{bmatrix}$$

THEN RREF TO FIND SOLUTION

$$-3(3y-5) - 2(x+4) = 2$$

$$-9y + 15 - 2x - 8 = 2$$

$$-9y - 2x = -5$$

$$-2x - 9y = -5 \quad : \text{ MUST HAVE STANDARD ORDER}$$

18. (-2, 1)

#19 Use Cramer's Rule to solve

$$3a + 5b - 2c = 13$$

$$4a + 7b - 11c = -8$$

$$8a - 2b + 9c = 93$$

FIND DETERMINANT OF VARIABLES

$$\begin{vmatrix} 3 & 5 & -2 \\ 4 & 7 & -11 \\ 8 & -2 & 9 \end{vmatrix} = -369$$

THEN

$$D_A = \begin{vmatrix} 13 & 5 & -2 \\ -8 & 7 & -11 \\ 93 & -2 & 9 \end{vmatrix}$$

$$D_A = -2952$$

$$D_B = \begin{vmatrix} 3 & 13 & -2 \\ 4 & -8 & -11 \\ 8 & 93 & 9 \end{vmatrix}$$

$$D_B = 369$$

$$D_C = \begin{vmatrix} 3 & 5 & 13 \\ 4 & 7 & -8 \\ 8 & -2 & 93 \end{vmatrix}$$

$$D_C = -1107$$

$$A = \frac{-2952}{-369} = 8$$

$$B = \frac{369}{-369} = -1$$

$$C = \frac{-1107}{-369} = 3$$

19. (8, -1, 3)

Section 5: Solving Word Problems with Matrices.

#20 "Mustangs Running Wild."

Those mighty Mustangs were on the run again with a dominating performance in the State sectional tournament for track and field. In all the hoopla the statisticians forgot to record the number of first, second, third, and fourth place finishes by individual event. (Note: Placing in any event is only recorded once regardless if there were multiple participants for the event. In other words, if the relay team for the 4 x 1600m placed it only counts as one place finisher for the team.)

Help sort out this mess, given the following facts. We know that are beloved Tuslaw Mustangs placed in a total of 32 events. Points were awarded as 10 pts, 7 pts, 4 pts, and 1 pt for each of first, second, third, and fourth place finishes in each event respectively. The Mustangs recorded a total of 212 points in the meet. Additionally, it is known that the number of first place finishes was eight less than the sum of second, third, and fourth place finishes. Finally, it was figured that three times the number of third place finishes plus four times the second places finishes was equal to five less than three times the difference of twice the number of first, and fourth place finishes. (Take your time when translating that last part my young scholars.) Obviously the final result should be integers.

FIRST SECOND THIRD FOURTH
W X Y Z

$$W + X + Y + Z = 32 \quad : \text{EVENTS PLACED IN}$$

$$10W + 7X + 4Y + Z = 212 \quad : \text{POINTS EARNED}$$

$$\text{FIRST} = (\text{SECOND} + \text{THIRD} + \text{FOURTH}) - 8$$

$$W = X + Y + Z - 8$$

$$W - X - Y - Z = -8$$

$$3[\text{THIRD}] + 4[\text{SECOND}] = 3[2 * \text{FIRST} - \text{FOURTH}] - 5$$

$$3Y + 4X = 3(2W - Z) - 5$$

$$3Y + 4X = 6W - 3Z - 5$$

$$-6W + 4X + 3Y + 3Z = -5$$

AUGMENTED MATRIX

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 32 \\ 10 & 7 & 4 & 1 & 212 \\ 1 & -1 & -1 & -1 & -8 \\ -6 & 4 & 3 & 3 & -5 \end{bmatrix}$$

THEN RREF TO FIND SOLUTIONS

20. (Write answers in box)

FIRST: 12

SECOND: 7

THIRD: 10

FOURTH: 3

#21

A certain company builds electric cars and places orders for **motors** and **controllers**. The orders come from two different distributors. Last month there was a total of 25 motors and 14 controllers shipped from the two distributors to the company. **Distributor A** sent a total 29 items while **distributor B** sent a total of 10 items.

A motor from distributor A costs \$650 while a motor from distributor B costs \$575. Controllers sent from distributor A cost \$525 while distributor B charges \$620. One may wonder why not just order parts from one distributor? There are logistical reasons for using multiple distributors such as shipping times or quality of parts. In either case, the parts overhead for last month totaled \$23,190. The manager misplaced the purchase orders and needs to identify how many of each part were ordered from the respective distributors. Help her out of this dilemma.

There is a solution to this problem so you may have to be selective with the system of equations used to solve the problem. Consider each part to have a unique variable when creating the system of equations.

DISTRIBUTOR	A	B	
MOTORS	w	x	25
CONTROLLERS	y	z	14
	29	10	39

$$\left. \begin{array}{l} w + x = 25 \\ y + z = 14 \\ w + y = 29 \\ x + z = 10 \end{array} \right\}$$

* USING ONLY THIS SYSTEM PRODUCES PARAMETRIC SOLUTIONS

CONSIDER PURCHASE EQUATION:

$$650w + 575x + 525y + 620z = 23,190$$

SO USE AUGMENTED MATRIX

1	1	0	0	25
0	0	1	1	14
1	0	1	0	29
650	575	525	620	23,190

* ANY OF FOUR DIFFERENT SYSTEMS COULD WORK.

21. (Write answers in box)

MOTORS FROM A = 17
MOTORS FROM B = 8
CONTROLLER FROM A = 12
CONTROLLER FROM B = 2