**First Quarter Exam** **Name** KEY

## Linear Equations

1. Write the equation of the line that has a slope of $m = -\frac{2}{7}$ that contains the point ( 2 , – 4 ) in **standard form**.

WITH $m = -\frac{2}{7}$

STANDARD FORM: $2x + 7y = C$

PLUG IN $(2,-4)$ $2(2) + 7(-4) = C$

$-24 = C$

SO $2x + 7y = -24$

1. $2x + 7y = -24$

2. Write the equation of the line that is **perpendicular** to $y = \frac{5}{2}x - 17\frac{2}{3}$ that contains the point ( 3 , 2 ) in **standard form**.

WITH $m = \frac{5}{2}$

NEED $m_{\perp} = -\frac{2}{5}$

STANDARD FORM $2x + 5y = C$

PLUG IN $(3,2)$ $2(3) + 5(2) = C$

$16 = C$

SO $2x + 5y = 16$

2. $2x + 5y = 16$

## Writing Step Functions.

3. Write the equation of the **floor** function that contains steps or *treads* that are 7 units **long** and will *rise* 3 units. The first step must start at the origin and run along the x-axis of the first quadrant. Be sure to use proper notation $\lfloor\ \rfloor$ or $\lceil\ \rceil$.

TO CREATE TREADS 7 UNITS LONG,
USE COEFFICIENT OF $1/7$ INSIDE FLOOR FUNCTION

TO RISE 3 UNITS, USE 3 OUTSIDE OF FLOOR FUNCTION

3. $y = 3\lfloor \tfrac{1}{7}x \rfloor$

4. **Restricted domains.** Create the steps that lead with the first tread from ( –3 , 0 ] or 3 units long and rise 6 units. The end of the stairs must be the x interval of (12, 15 ]. *Look at notation, what would be appropriate a ceiling or floor function? Test your equation to ensure that the conditions are being met.*

USE A CEILING FUNCTION

TO CREATE TREADS 3 UNITS LONG,
USE COEFFICIENT OF $1/3$ INSIDE CEILING FUNCTION

4. $y = 6\lceil \tfrac{1}{3}x \rceil \quad \{-3 < x \leq 15\}$

TO RISE 6 UNITS, USE 6 OUTSIDE OF CEILING FUNCTION

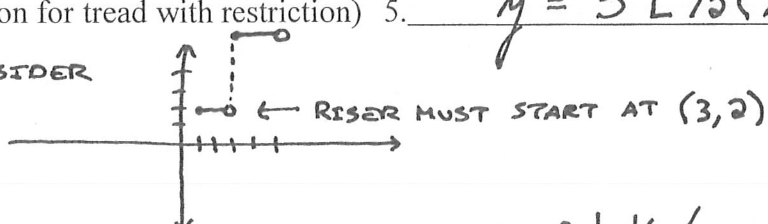
STEPS MUST RUN FROM DOMAIN $(-3, 15]$ SO RESTRICT DOMAIN

5. Write the set of **floor functions** using **phase shifts** and restricted domains that create **treads and risers** with the following characteristics. A set of **six** steps starting at a tread length of 2 units at the point ( 1, 2 ) then rising 5 units. *There should be six treads and five risers, be careful as the function for the risers will NOT have the same phase shift generated by the point (1,2) because the tread shifts the graph more to the right to start the risers.*

TREAD = COEFFICIENT $\frac{1}{2}$ INSIDE STEP FUNCTION
RISER = 5 OUTSIDE STEP FUNCTION
USE PHASE SHIFT FOR $(1,2)$

(Equation for tread with restriction) 5. $y = 5\lfloor \tfrac{1}{2}(x-1) \rfloor + 2 \quad \{1 \leq x < 13\}$

CONSIDER




(Equation for riser with restriction) 5. $x = 2\lfloor \tfrac{1}{5}(y-2) \rfloor + 3 \quad \{2 \leq y \leq 27\}$

WHEN WRITING EQUATION SOLVE FOR $x$

SO LENGTH OF RISER WILL PUT $1/5$ INSIDE STEP FUNCTION WITH 2 OUTSIDE.

THEN REPRESENT PHASE SHIFT FOR POINT $(3,2)$

## Writing Linear Absolute Value Functions

6. Write an absolute value equation where the line of symmetry is located at x = 5 that has been shifted up three units and opens down with a slope of $\frac{4}{5}$. Be sure to use proper absolute value notation $| \, |$.

LINE of SYMMETRY of $x=5$
MEANS PHASE SHIFT $(x-5)$
SHIFTED UP THREE UNITS
MEANS PHASE SHIFT $(y-3)$

6. $y = -\frac{4}{5}|x-5| + 3$

7. Write an absolute value equation where the line of symmetry is located at x = – 3 that has been shifted down five units and opens up with a slope of 3. Be sure to use proper absolute value notation $| \, |$.

* THERE ARE ALTERNATE FORMS THAT COULD WORK FOR THESE FUNCTIONS

7. $y = 3|x+3| - 5$

8. Write an absolute value equation where the line of symmetry is located at y = –2 that has been shifted to the left four units and opens right with an actual slope of $\frac{2}{3}$. Be sure to use proper absolute value notation $| \, |$.

* NEEDS TO BE SOLVED FOR $x$
WITH LINE of SYMMETRY AT $y=-2$

8. $x = \frac{3}{2}|y+2| - 4$

* SLOPE WOULD BE RECIPROCATED WHEN EQUATION IS SOLVED FOR $x$

9. Write an absolute value equation where the line of symmetry is located at y = 11 that has been shifted to the right two units and opens left with a slope of 5. Be sure to use proper absolute value notation $| \, |$.

9. $x = \frac{-1}{5}|y-11| + 2$

* ALTERNATE FORMS

#6 $(y-3) = -\left|\frac{4}{5}(x-5)\right|$

#7 $(y+5) = |3(x+3)|$

#8 $(x+4) = \left|\frac{3}{2}(y+2)\right|$

#9 $(x-2) = -\left|\frac{1}{5}(y-11)\right|$

## Writing Parabolic Equations

10. Write a parabolic equation where the line of symmetry is located at x = 2 that has been shifted down thirteen units and opens up with a standard stretch. Be sure to use proper $y = a(x-h)^2 + k$

↓

NO OTHER FORM ACCEPTED

10. $y = (x-2)^2 - 13$

11. Write a parabolic equation with a vertex at ( 3 , – 8 ) that has movement from the vertex as follows. Move right or left two units then up two units. Move right or left four units then up eight units. Finally, move right or left six units from the vertex then up eighteen units. Be sure to use proper $y = a(x-h)^2 + k$

11. $y = \frac{1}{2}(x-3)^2 - 8$

12. Consider the **standard form** of a parabola $y = Ax^2 + Bx + C$, Find the ( h , k ) form if the parabola contains the points ( 10 , 25 ) , ( 1, – 2 ), and ( 4 , 1 ). *Three equations – three unknowns, then complete the square.*

* SEE NEXT PAGE FOR WORK

12. $y = \frac{1}{3}(x-1)^2 - 2$

13. Consider the **standard form** of a parabola $y = Ax^2 + Bx + C$, Find the ( h , k ) form if the parabola contains the points ( 3 , 1 ) , ( 7 , 13 ), and ( – 2 , $-2\frac{3}{4}$ ). Hint: Three equations three unknowns, then complete the square.

* SEE NEXT PAGES FOR WORK

13. $y = \frac{1}{4}(x+1)^2 - 3$

14. Write a parabolic equation where the line of symmetry is located at y =– 6 that has been shifted to the left four units and opens to the left with the horizontal stretch of 3 times the standard movement. Be sure to use proper $x = a(y-k)^2 + h$

* THE EFFECTS of a ARE UNIVERSAL ON PARABOLAS
STANDARD ALWAYS = 1
OBTUSE ALWAYS $0 < |a| < 1$
ACUTE ALWAYS $|a| > 1$

14. $x = -3(y+6)^2 - 4$

15. Write a parabolic equation with a vertex at ( 5 , –3 ) that has movement from the vertex as follows. Move up or down five units then left five units. Move up or down ten units then left twenty units. Finally, move up or down fifteen units from the vertex then left forty five units. Be sure to use proper $x = a(y-k)^2 + h$

15. $x = -\frac{1}{5}(y+3)^2 + 5$

#12 $Ax^2+Bx+C=y$

CONSIDER

$(10,25)$: $100A+10B+C=25$

$(1,-2)$: $1A+B+C=-2$

$(4,1)$: $16A+4B+C=1$

ENTER AUGMENTED MATRIC IN CALCULATOR

THEN USED RREF

So $A=1/3$, $B=-2/3$, AND $C=-5/3$

NOW $y=\frac{1}{3}x^2-\frac{2}{3}x-\frac{5}{3}$

$3y=x^2-2x-5$

$3y=[x^2-2x+1]-5-1$ : BALANCE EQUATION

$\frac{3y}{3}=\frac{(x-1)^2}{3}-\frac{6}{3}$

$y=1/3(x-1)^2-2$

#13 $Ax^2+Bx+C=y$

CONSIDER

$(3,1)$: $9A+3B+C=1$

$(7,13)$: $49A+7B+C=13$

$(-2,-\frac{11}{4})$ $4A-2B+C=-\frac{11}{4}$

ENTER AUGMENTED MATRIX

THEN SOLVE WITH RREF OPTION

SO $A=1/4$, $B=1/2$, AND $C=-11/4$

NOW $y=\frac{1}{4}x^2+\frac{1}{2}x-\frac{11}{4}$

$4y=x^2+2x-11$

$4y=[x^2+2x+1]-11-1$ : BALANCE EQUATION

$\frac{4y}{4}=\frac{(x+1)^2}{4}-\frac{12}{4}$

$y=1/4(x+1)^2-3$

## Writing Equations for Ellipses

16. Write an elliptical equation with a center at ( – 8 , –19 ) that has a major axis length of twenty two parallel to the **y-axis** and a minor axis length of ten. Be sure to use proper

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

16. $\dfrac{(x+8)^2}{25}+\dfrac{(y+19)^2}{121}=1$

17. Write an elliptical equation with a center at ( 6 , – 5 ) that has a major axis length of sixteen parallel to the **x-axis** and a minor axis length of two. Be sure to use proper

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

17. $\dfrac{(x-6)^2}{64}+\dfrac{(y+5)^2}{1}=1$

18. Use completing the square techniques to convert this standard form equation into the (h , k) form. $12x^2-96x+7y^2+14y+5=-110$ *When writing equations one may want to remember that $a^2$ and $b^2$ are not always the squares of integer values. In other words, sometimes students are expected to work with less than ideal equations.*

* WORK ON NEXT PAGE

18. $\dfrac{(x-4)^2}{7}+\dfrac{(y+1)^2}{12}=1$

Use completing the square techniques to convert this standard form equation into the (h , k) form. $108x^2-108x+288y^2+384y=709$ **Please remember that not all ellipses have lattice points for centers.** Additionally $a^2$ and $b^2$ are not always the squares of integer values. This may be a question to leave on the table and finish when the rest of the test is finished. **Do not let it deplete all of your time.**

* WORK ON NEXT PAGE

$\dfrac{(x-\frac{1}{2})^2}{8}+\dfrac{(y+\frac{2}{3})^2}{3}=1$

\#18 $12x^2 - 96x + 7y^2 + 14y + 5 = -110$

$12x^2 - 96x + 7y^2 + 14y = -115$ : MOVE FROM LEFT SIDE

$12[x^2 - 8x + 16] + 7[y^2 + 2y + 1] = -115 + 192 + 7$

$$\frac{12(x-4)^2}{84} + \frac{7(y+1)^2}{84} = \frac{84}{84}$$

$$\frac{(x-4)^2}{7} + \frac{(y+1)^2}{12} = 1$$

$108x^2 - 108x + 288y^2 + 384y = 709$

$108x^2 - 108x + 288y^2 + 384y = 709$

$108[x^2 - x + \tfrac{1}{4}] + 288[y^2 + \tfrac{4}{3}y + \tfrac{4}{9}] = 709 + 27 + 128$

$$\frac{108(x-\frac{1}{2})^2}{864} + \frac{288(y+\frac{2}{3})^2}{864} = \frac{864}{864}$$

$$\frac{(x-\frac{1}{2})^2}{8} + \frac{(y+\frac{2}{3})^2}{3} = 1$$

## Writing Equations for Circles

19. Write the equation of the circle with a center at ( –9 , 11 ) and a radius of length of seventeen. Be sure to use proper $(x-h)^2+(y-k)^2=r^2$

19. $(x+9)^2+(y-11)^2=289$

20. Use completing the squares techniques to convert this standard form equation into the (h , k) form. $5x^2+70x+5y^2-50y=-290$ Please remember that not all circles have lattice points for centers. Additionally $r^2$ does not always represent the square of an integer value. In other words, sometimes students are expected to work with less than ideal equations.

20. $(x+7)^2+(y-5)^2=16$

21. Consider the **general form** of a circle $x^2+Ax+y^2+By+C=0$, Find the ( h , k ) form if the circle contains the points ( 3 , 6 ) , ( 7 , 8 ), and ( 10 , 7 ). Hint: Three equations three unknowns, then complete the squares.

21. $(x-7)^2+(y-3)^2=25$

## Writing Equations of Hyperbolas

22. Write the equation of the horizontal hyperbola with a center at ( 8 , – 3 ) with slant asymptotes with slopes of $\frac{1}{3}$ and $-\frac{1}{3}$.

22. $\frac{(x-8)^2}{9}-\frac{(y+3)^2}{1}=1$

23. Write the equation of the vertical hyperbola with a center at ( –5 , –2 ) with slant asymptotes with slopes of $\frac{3}{11}$ and $-\frac{3}{11}$.

23. $\frac{(y+2)^2}{9}-\frac{(x+5)^2}{121}=1$

#20 $5x^2 + 70x \quad + 5y^2 - 50y \quad = -290$

$5[x^2 + 14x + 49] + 5[y^2 - 10y + 25] = -290 + 245 + 125$

$\frac{5(x+7)^2}{5} + \frac{5(y-5)^2}{5} = \frac{80}{5}$

$(x+7)^2 + (y-5)^2 = 16$

#21 CONSIDER EACH POINT, CREATE AN EQUATION WITH THREE UNKNOWNS

$x^2 + Ax + y^2 + By + C = 0$

$(3,6)$: $9 + 3A + 36 + 6B + C = 0$

$3A + 6B + C = -45$

$(7,8)$: $49 + 7A + 64 + 8B + C = 0$

$7A + 8B + C = -113$

$(10,7)$: $100 + 10A + 49 + 7B + C = 0$

$10A + 7B + C = -149$

ENTER AUGMENTED MATRIX THEN USE RREF OPTION TO SOLVE SYSTEM

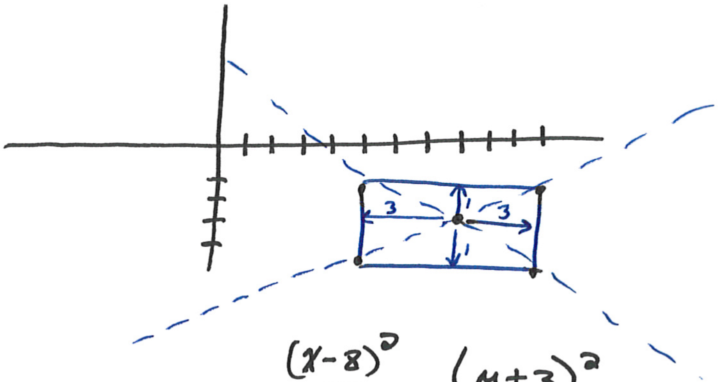
So $A = -14$, $B = -6$, AND $C = 33$

$x^2 - 14x \quad + y^2 - 6y \quad = -33$ : MOVE C TO OTHER SIDE OF EQUATION

$x^2 - 14x + 49 + y^2 - 6y + 9 = -33 + 49 + 9$

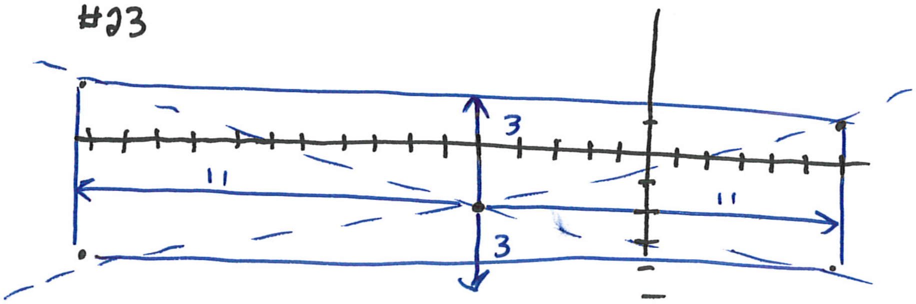
$(x-7)^2 + (y-3)^2 = 25$

#22




$$\frac{(x-8)^2}{9} - \frac{(y+3)^2}{1} = 1$$

#23



## Writing Equations of Trig Equations

24. Write the equation of a sine function that has an amplitude of seven and a period of three that was been shifted five units up and two units to the right.

$y = a\sin(bx) + c$

24. $y = 7\sin\left(\frac{2\pi}{3}(x-2)\right) + 5$

AMPLITUDE = 7, SO $a = 7$

PERIOD OF 3, SO $\frac{2\pi}{b} = 3$ AND $b = \frac{2\pi}{3}$

PHASE SHIFT: $(2,5)$ * PHASE SHIFTS ARE UNIVERSAL AMONG EQUATIONS

25. Write the equation of a cosine function that has an amplitude of $\frac{3}{5}$ and a period of $8\pi$ that was been shifted three units down and one unit to the left.

$y = a\cos(bx) + c$

25. $y = \frac{3}{5}\cos\left(\frac{1}{4}(x+1)\right) - 3$

AMPLITUDE = $\frac{3}{5}$, SO $a = \frac{3}{5}$

PERIOD OF $8\pi$, SO $\frac{2\pi}{b} = 8\pi$ AND $b = 1/4$

PHASE SHIFT $(-1,-3)$

## Writing Equations Using Rotations

26. Write the equation of $\frac{(x-5)^2}{36} + \frac{(y+7)^2}{121} = 1$ rotated 60° clockwise. *Be sure to write neat as the placement of parentheses is crucial.*

CONVERT 60° TO RADIANS = $\pi/3$

26. $\frac{\left[(x-5)\cos(\pi/3) - (y+7)\sin(\frac{\pi}{3})\right]^2}{36} + \frac{\left[(x-5)\sin(\frac{\pi}{3}) + (y+7)\cos(\frac{\pi}{3})\right]^2}{121} = 1$

* CRAZY BUT JUST SUBSTITUTION

27. Write the equation of $y + 1 = (x+4)^2$ rotated 120° clockwise. *Be sure to write neat as the placement of parentheses is crucial.*

CONVERT 120° TO RADIANS = $\frac{2}{3}\pi$

27. $(x+4)\sin\left(\frac{2\pi}{3}\right) + (y+1)\cos\left(\frac{2\pi}{3}\right) = \left[(x+4)\cos\left(\frac{2\pi}{3}\right) - (y+1)\sin\left(\frac{2\pi}{3}\right)\right]^2$

## Even, Odd or Neither Functions

Determine whether each function is even, odd, or neither.

28. $g(x) = 3x$ 28. ODD

29. $h(x) = x^2 + \cos(x^3)$ 29. EVEN

30. $w(x) = 2x - 3$ 30. NEITHER

31. $m(x) = x^3 - 4x$ 31. ODD

32. $f(x) = |3x| - 2$ 32. EVEN

33. $q(x) = 2x^5 - 3x^3 + 7x$ 33. ODD

34. $d(x) = \frac{2}{3}(x-3)^2 + 4$ 34. NEITHER

35. $a(x) = 4 \cdot \sin(3\pi \cdot x)$ 35. ODD

* PLUG INTO DEFINITION AND GRAPH