Take Home Portion (*Helping Videos Online*) Name KEY

Solve each of the inequalities

#1 Solve the inequality, be sure answer is in **interval notation.**

$$\frac{2x^2+x-15}{x^2+3x-28}\le 0$$

#1 $(-7,-3]\cup[2\frac{1}{2},4)$

#2 Solve the inequality, be sure answer is in **interval notation.**

$$\frac{g^2+2g-35}{g^2+8g+12}\le 0$$

#2 $[-7,-6)\cup(-2,5]$

3 Graph and state the domain and range using **interval notation.**

a) $y=-2x^4+7x^3+2x^2-11x-7$

3a Domain: $(-\infty,\infty)$

3a Range: $(-\infty, 9.563]$

b) $y=\dfrac{x^2-3x-10}{x^2-5x+4}$

3b Domain: $(-\infty,1)\cup(1,4)\cup(4,\infty)$

3b Range: $(-\infty, 1.114]\cup[4.886,\infty)$

c) $y=\left|-(x-3)^2+7\right|-3$

3c Domain: $(-\infty,\infty)$

3c Range: $[-3,\infty)$

*The **maximum height** or greatest **y–value** of the function is not the same as the **extreme behavior** of the function. Use a graphing utility to explore this function. When the graph of the function is on the screen use the 2nd Calc option, select maximum, then select appropriate left and right bounds to isolate the suspected maximum. Finally the calculator will prompt for a guess within the designated boundaries and report the maximum value for the function given the domain it was to search under.

#4 Write each of the following in standard H , K forms.

a. $9x^2+9y^2-90x-6y+190=0$

#4a $(x-5)^2+(y-1/3)^2=4$

b. $9x^2+4y^2-18x-16y-11=0$

#4b $\frac{(x-1)^2}{4}+\frac{(y-2)^2}{9}=1$

c. $y^2-2x+6y+1=0$

#4c $x=\frac{1}{2}(y+3)^2-4$

#5 Solve

a) $4|2x+5|^2-11|2x+5|=-6$

#5a $x=\{-3\frac{1}{2},-2\frac{7}{8},-2\frac{1}{8},-1\frac{1}{2}\}$

b) $3|5x-2|^2-14|5x-2|=5$

#5b $x=\{-\frac{3}{5},1\frac{2}{5}\}$

* Answers need to be written as fractions or mixed numbers. However, if one wanted to check answers by graphing the function that would be acceptable. Procedure – set each equation equal to zero, in other words move the constant from the right to the left hand side using the appropriate operation of addition or subtraction. Next graph the expression using the equation editor on the calculator. Be sure to call up the absolute value function (MATH button, select NUM, choose option 1) Finally, use the 2nd calc option and select zero. Then follow the on–screen instructions to find the zeros. Those results should match the solutions arrived at above.

#6 Find the solution set for $g(x) = (x-5)^{\frac{1}{3}} + 5$ and $g^{-1}(x)$

#6 $x = \{4, 5, 6\}$

#7 Find the solution set for $f(x) = 2 - (x-3)^{\frac{1}{3}}$ and $f^{-1}(x)$

#7 $x \approx \{1.46, 2, 2.68, 3, 3.15\}$

Identify zeroes, all asypmtotes, and perform a sign check, remember to check extreme values.

#8. $y = \dfrac{2x^2 - 7x - 15}{x + 4}$

zeroes $x = \{-1\frac{1}{2}, 5\}$

vertical asymptotes $x \neq -4$

slant asymptotes $y = 2x - 15$

Identify zeroes, all asypmtotes, and perform a sign check, remember to check extreme values.

#9. $y = \dfrac{4x^2 + 11x - 3}{x - 2}$

zeroes $x = \{-3, \frac{1}{4}\}$

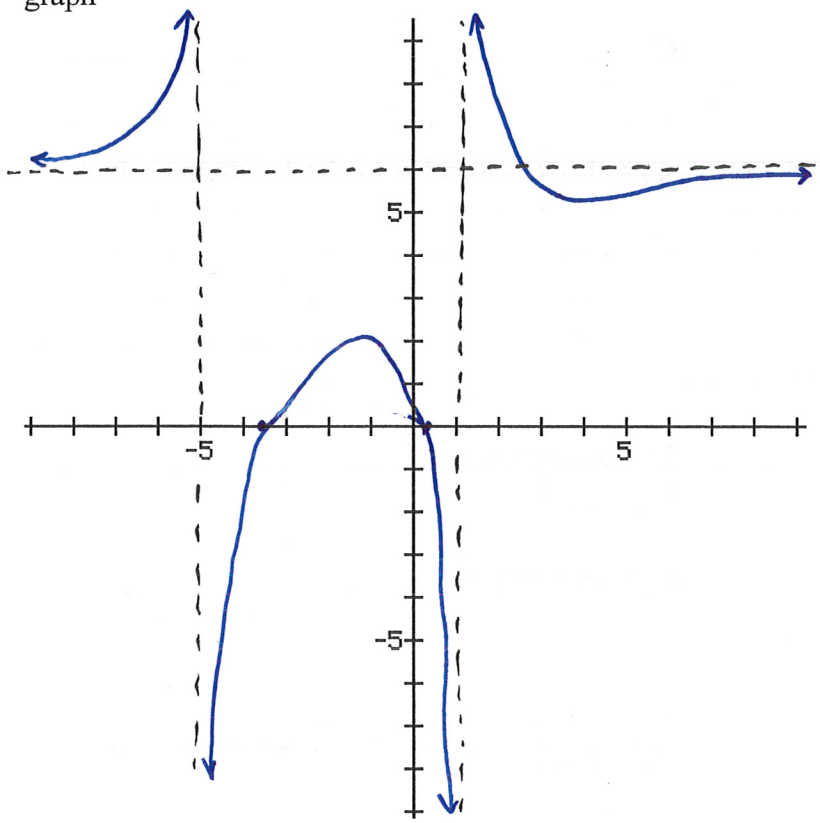
vertical asymptotes $x \neq 2$

slant asymptotes $y = 4x + 19$

Graph and identify zeroes, all asypmtotes, and perform a sign check, remember to check extreme values.

10. $y = \dfrac{6x^2+19x-7}{x^2+4x-5}$ $\quad \dfrac{(3x-1)(2x+7)}{(x-1)(x+5)}$

graph



zeroes $x = \{-3\frac{1}{2}, \frac{1}{3}\}$

vertical asymptotes $x \neq \{-5, 1\}$

horizontal asymptotes $y = 6$

$\lim_{x \to -\infty} f(x) = 6^+$ $\quad \lim_{x \to \infty} f(x) = 6^-$

sign check (+) DNE (−) 0 (+) 0 (−) DNE (+)

$-5 \quad -3\frac{1}{2} \quad \frac{1}{3} \quad 1$

10b Domain: $(-\infty, -5) \cup (-5, 1) \cup (1, \infty)$ (Interval notation)

10c Range: $(-\infty, 2.534] \cup [5.800, \infty)$ (Interval notation)

#1 $\dfrac{2x^2+x-15}{x^2+3x-28} \leq 0$

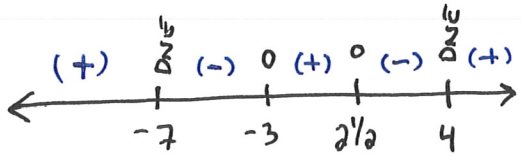
$\dfrac{(2x-5)(x+3)}{(x+7)(x-4)}$

DOMAIN FOR ≤ 0

$(-7,-3] \cup [2\frac{1}{2}, 4)$

ZEROES $x = \{-3, 2\frac{1}{2}\}$

RESTRICTIONS $x \neq \{-7, 4\}$



#2 $\dfrac{g^2+2g-35}{g^2+8g+12} \leq 0$

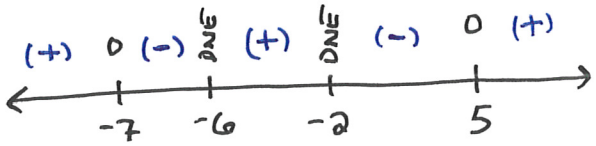
$\dfrac{(g+7)(g-5)}{(g+6)(g+2)} \leq 0$

DOMAIN FOR ≤ 0

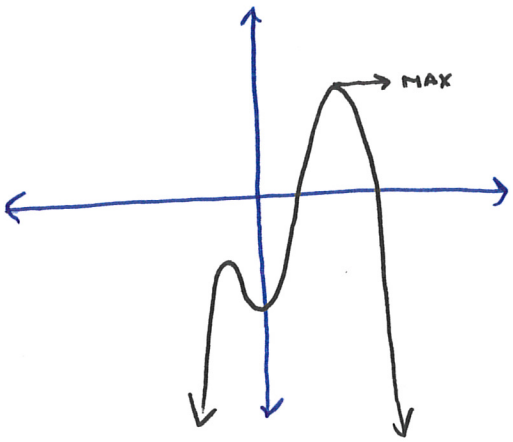
$[-7,-6) \cup (-2,5]$

ZEROES $x = \{-7, 5\}$

RESTRICTIONS $x \neq \{-6, -2\}$



#3A SKETCH FROM CALCULATOR

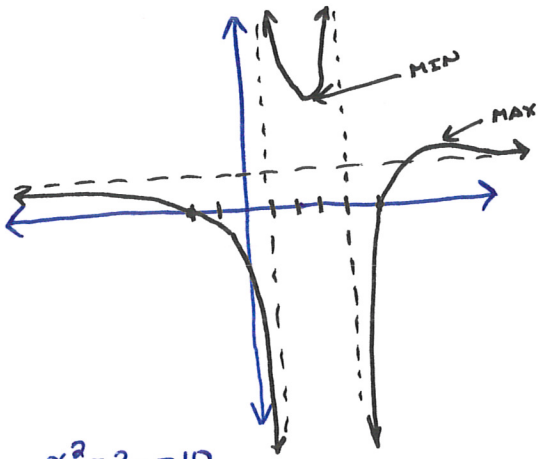


DOMAIN: $(-\infty, \infty)$

* NO RESTRICTED VALUES FOR POLYNOMIALS

RANGE: $(-\infty, 9.563]$

#3B SKETCH FROM CALCULATOR



DOMAIN: $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$

RANGE: $(-\infty, 1.114] \cup [4.886, \infty)$

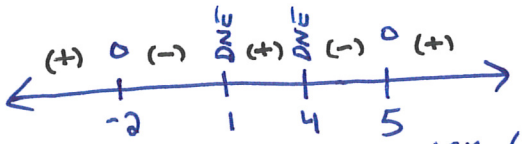
$$y = \frac{x^2 - 3x - 10}{x^2 - 5x + 4}$$

$$y = \frac{(x-5)(x+2)}{(x-4)(x-1)}$$

ZEROES $x = \{-2, 5\}$

HORIZONTAL ASYMPTOTE $y = 1$

RESTRICTIONS $x \neq \{1, 4\}$

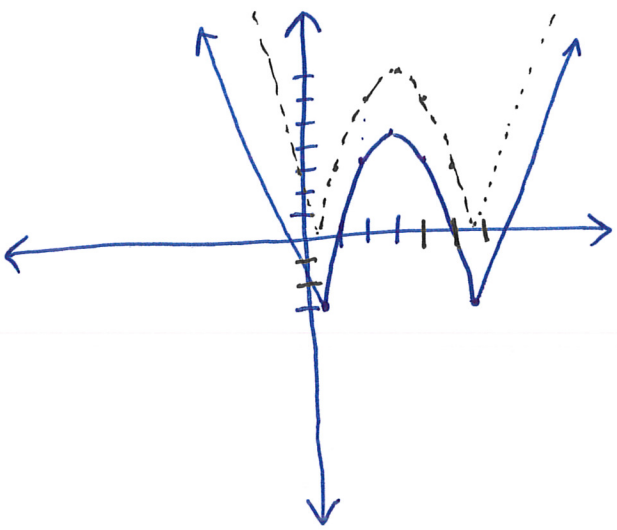


$\lim_{x \to -\infty} f(x) = 1^-$

$\lim_{x \to \infty} f(x) = 1^+$

#3c $y = \left| -(x-3)^2 + 7 \right| - 3$

SKETCH



DOMAIN: $(-\infty, \infty)$

RANGE: $[-3, \infty)$

#4A $9x^2 + 9y^2 - 90x - 6y + 190 = 0$

$9x^2 - 90x + 9y^2 - 6y + 190 = 0$

$9[x^2 - 10x + 25] + 9[y^2 - \frac{2}{3}y + \frac{1}{9}] = -190 + 225 + 1$

$9(x-5)^2 + 9(y-1/3)^2 = 36$

$(x-5)^2 + (y-1/3)^2 = 4$

#4B $9x^2 + 4y^2 - 18x - 16y - 11 = 0$

$9x^2 - 18x + 4y^2 - 16y = 11$

$9[x^2 - 2x + 1] + 4[y^2 - 4y + 4] = 11 + 9 + 16$

$9(x-1)^2 + 4(y-2)^2 = 36$

$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{9} = 1$

#4C $y^2 - 2x + 6y + 1 = 0$

$[y^2 + 6y\ ] + 1 = 2x$

$[y^2 + 6y + 9] + 1 - 9 = 2x$

$(y+3)^2 - 8 = 2x$

$\frac{1}{2}(y+3)^2 - 4 = x$

#5A $4|2x+5|^2 - 11|2x+5| + 6 = 0$

LET $F = |2x+5|$

$4F^2 - 11F + 6 = 0$ $\quad AC = 24$ $\quad B = -11$

$-8 \quad -3$

$(4F - 3)(F - 2) = 0$

$F = 3/4 \quad F = 2$

So $|2x+5| = 3/4 \quad |2x+5| = 2$

$2x+5 = 3/4$ OR $2x+5 = -3/4$ OR $2x+5 = 2$ OR $2x+5 = -2$

$2x = 3/4 - 5$ $\quad 2x = -3/4 - 5$ $\quad 2x = 2 - 5$ $\quad 2x = -2 - 5$

$\frac{2x}{2} = \frac{-4\frac{1}{4}}{2}$ $\quad \frac{2x}{2} = \frac{-5\frac{3}{4}}{2}$ $\quad \frac{2x}{2} = \frac{-3}{2}$ $\quad \frac{2x}{2} = \frac{-7}{2}$

$x = -2\frac{1}{8}$ $\quad x = -2\frac{7}{8}$ $\quad x = -1\frac{1}{2}$ $\quad x = -3\frac{1}{2}$

$x = \{-3\frac{1}{2}, -2\frac{7}{8}, -2\frac{1}{8}, -1\frac{1}{2}\}$

#5B $3|5x-2|^2 - 14|5x-2| - 5 = 0$

LET $M = |5x-2|$

$3M^2 - 14M - 5 = 0$ $\quad AC = -15$ $\quad B = -14$

$-15 \quad 1$

$(3M + 1)(M - 5) = 0$

$M = -1/3 \quad M = 5$

So $|5x-2| = -1/3 \quad |5x-2| = 5$

NO SOLUTION

$5x - 2 = 5$ OR $5x - 2 = -5$

$5x = 5 + 2$ $\quad 5x = -5 + 2$

$\frac{5x}{5} = \frac{7}{5}$ $\quad \frac{5x}{5} = \frac{-3}{5}$

$x = 1\frac{2}{5}$ $\quad x = -3/5$

$x = \{-3/5, 1\frac{2}{5}\}$

#6 $g(x) = (x-5)^{1/3} + 5$

$g(x)$: $y = (x-5)^{1/3} + 5$

$g^{-1}(x)$: $x = (y-5)^{1/3} + 5$

$x - 5 = (y-5)^{1/3}$

$(x-5)^3 = y - 5$

$g^{-1}(x)$: $(x-5)^3 + 5 = y$

TO FIND SOLUTION SET

$(x-5)^{1/3} + 5 = (x-5)^3 + 5$

$-5 \qquad\qquad -5$

$(x-5)^{1/3} = (x-5)^3$

$0 = (x-5)^3 - (x-5)^{1/3}$

LET $P = (x-5)^{1/3}$

SO $0 = P^9 - P$

AND $0 = P(P^8 - 1)$ } USE PLYSMTH TO FIND ROOTS FOR $P^8 - 1$

$P = 0 \quad P = -1 \quad P = 1$

$0 = \sqrt[3]{x-5}$	$-1 = \sqrt[3]{x-5}$	$1 = \sqrt[3]{x-5}$
$0 = x - 5$	$-1 = x - 5$	$1 = x - 5$
$5 = x$	$-1 + 5 = x$	$1 + 5 = x$
	$4 = x$	$6 = x$

$x = \{4, 5, 6\}$

#7 $f(x)$: $y = 2 - (x-3)^{1/3}$

$f^{-1}(x) \Rightarrow$ $x = 2 - (y-3)^{1/3}$

$x - 2 = -(y-3)^{1/3}$

$2 - x = (y-3)^{1/3}$

$(2-x)^3 = y - 3$

$f^{-1}(x)$: $3 + (2-x)^3 = y$

TO FIND SOLUTION SET

$2 - (x-3)^{1/3} = 3 + (2-x)^3$

$-2 \qquad\qquad -2$

$0 = 1 + (2-x)^3 + (x-3)^{1/3}$

LET $E = (x-3)^{1/3}$

$E^3 = x - 3$

$E^3 + 3 = x$

$0 = 1 + (2 - (E^3+3))^3 + E$

$0 = 1 + (-E^3 - 1)^3 + E$

$0 = 1 - E^9 - 3E^6 - 3E^3 - 1 + E$

$0 = -E^9 - 3E^6 - 3E^3 + E$

$0 = -E(E^8 + 3E^5 + 3E^2 - 1)$

$E = 0 \quad E = -1 \quad E = -1.154 \quad E = -.682 \quad E = .536$

$E^3 + 3 = x$

$0^3 + 3 = x$	$-1^3 + 3 = x$	$(-1.154)^3 + 3 = x$	$(-.682)^3 + 3 = x$	$(.536)^3 + 3 = x$
$3 = x$	$-1 + 3 = x$			
	$2 = x$	$1.463 = x$	$2.683 = x$	$3.154 = x$

$x \approx \{1.46, 2, 2.68, 3, 3.15\}$

#8 $y = \dfrac{2x^2 - 7x - 15}{x+4}$

$y = \dfrac{(2x+3)(x-5)}{x+4}$

ZEROES $x = \{-1\frac{1}{2}, 5\}$

RESTRICTION $x \neq -4$ VERTICAL ASYMPTOTE

SLANT ASYMPTOTE $y = 2x - 15$

$$
\begin{array}{r}
2x - 15 \\
x+4 \overline{)\,2x^2 - 7x - 15} \\
-2x^2 + 8x \\
\hline
-15x - 15 \\
+15x + 60 \\
\hline
45
\end{array}
$$

#9 $y = \dfrac{4x^2 + 11x - 3}{x-2}$

$y = \dfrac{(4x-1)(x+3)}{x-2}$

ZEROES $x = \{-3, \frac{1}{4}\}$

RESTRICTION $x \neq 2$ VERTICAL ASYMPTOTE

SLANT ASYMPTOTE $y = 4x + 19$

$$
\begin{array}{r}
4x + 19 \\
x-2 \overline{)\,4x^2 + 11x - 3} \\
-4x^2 + 8x \\
\hline
19x - 3 \\
-19x + 38 \\
\hline
35
\end{array}
$$