SECTION B

PART I DETERMINE ODD FUNCTION $f(-x) = -f(x)$

#1 L.H.S. $f(-x) = [-x]^3 - 5[-x]$

$= -x^3 + 5x$

$y = x^3 - 5x$

R.H.S. $-f(x) = -[x^3 - 5x]$

$= -x^3 + 5x$

SINCE R.H.S. = L.H.S.

FUNCTION MUST BE ODD

#2 L.H.S. $f(-x) = [-x]^5 - 4[-x]^3 - [-x] + 1$

$= -x^5 + 4x^3 + x + 1$

$y = x^5 - 4x^3 - x + 1$

R.H.S. $-f(x) = -x^5 + 4x^3 + x - 1$

SINCE R.H.S. $\neq$ L.H.S

FUNCTION IS NOT ODD

#3 L.H.S. $f(-x) = \dfrac{[-x]^4 - 3[-x]^2 + 4}{[-x]}$

$= \dfrac{x^4 - 3x^2 + 4}{-x}$

$y = \dfrac{x^4 - 3x^2 + 4}{x}$

R.H.S. $-f(x) = -\left[\dfrac{x^4 - 3x^2 + 4}{x}\right]$

$= \dfrac{x^4 - 3x^2 + 4}{-x}$

SINCE R.H.S. = L.H.S.

FUNCTION MUST BE ODD

#4 L.H.S. $f(-x) = 3[-x] - \frac{5}{[-x]}$

$= -3x + \frac{5}{x}$

R.H.S. $-f(x) = -\left[3x - \frac{5}{x}\right]$

$= -3x + \frac{5}{x}$

$y = 3x - \frac{5}{x}$

SINCE R.H.S. = L.H.S.

FUNCTION MUST BE ODD

PART II DETERMINE EVEN FUNCTION $f(x) = f(-x)$

#1 L.H.S. $f(x) = x^4 - 5x^2 + 7$

R.H.S. $f(-x) = [-x]^4 - 5[-x]^2 + 7$

$= x^4 - 5x^2 + 7$

SINCE R.H.S = L.H.S

FUNCTION MUST BE EVEN

#2 L.H.S. $f(x) = 3|x|^2 - |5x| - 7$

R.H.S. $f(-x) = 3|[-x]|^2 - |5[-x]| - 7$: $|-x| = |x|$

$= 3|x|^2 - |5x| - 7$

SINCE R.H.S. = L.H.S.

FUNCTION MUST BE EVEN

#3 L.H.S. $f(x) = 3x - \frac{5}{x}$

R.H.S. $f(-x) = 3[-x] - \frac{5}{[-x]}$

$= -3x + \frac{5}{x}$

SINCE L.H.S. $\neq$ R.H.S.

FUNCTION CANNOT BE EVEN

#4 L.H.S. $f(x) = |x^3 - 5x|$

R.H.S. $f(-x) = |[-x]^3 - 5[-x]|$

$= |-x^3 + 5x|$

$= |-(x^3 - 5x)|$: CONSIDER $|-7| \overset{?}{=} |7|$

SINCE L.H.S. = R.H.S

<u>FUNCTION MUST BE EVEN</u>

PART III DETERMINE WHETHER FUNCTIONS ARE INVERSES. SHOW THAT BOTH $f(f^{-1}(x))$ AND $f^{-1}(f(x))$ EQUAL x

#1 $f(x): y = x^2 - 8x + 5$

$f^{-1}(x): y = 4 - \sqrt{x+11}$

$$f(f^{-1}(x)) = [4 - \sqrt{x+11}]^2 - 8[4 - \sqrt{x+11}] + 5$$

$$= 16 - 8\sqrt{x+11} + x + 11 \quad -32 + 8\sqrt{x+11} + 5$$

$$= \underline{27} + x \underline{\underline{- 8\sqrt{x+11}}} \quad \underline{-27} + \underline{\underline{8\sqrt{x+11}}}$$

$$= x$$

$$f^{-1}(f(x)) = 4 - \sqrt{[x^2 - 8x + 5] + 11}$$

$$= 4 - \sqrt{x^2 - 8x + 16}$$

$$= 4 - \sqrt{(x-4)^2}$$

$$= 4 - [x - 4]$$

$$= 4 - x + 4$$

$$= -x + 8$$

$\therefore$ NOT INVERSES

#2 $f(x)$: $y = \frac{x+7}{3}$

$f^{-1}(x)$: $y = 3x + \frac{7}{3}$

$$f(f^{-1}(x)) = \frac{\left[3x + \frac{7}{3}\right] + 7}{3}$$

$$= \frac{3x + \frac{7}{3} + \frac{21}{3}}{3}$$

$$= \frac{3x + \frac{28}{3}}{3}$$

$$= x + \frac{28}{9}$$

MUST EQUAL x, SO NO NEED TO MOVE TO PART 2

$\therefore$ NOT INVERSES

#3 $f(x)$: $y = x^3 - 15x^2 + 75x - 125$

$f^{-1}(x)$: $y = \sqrt[3]{x} + 5$

$$f(f^{-1}(x)) = \left[\sqrt[3]{x}+5\right]^3 - 15\left[\sqrt[3]{x}+5\right]^2 + 75\left[\sqrt[3]{x}+5\right] - 125$$

$$= \left[x + 15x^{2/3} + 75x^{1/3} + 125\right] \underline{-15x^{2/3} - 150x^{1/3} - 375} + \left[75x^{1/3} + 375\right] - 125$$

$$= x + \underline{15x^{2/3} - 15x^{2/3}} + \underline{\underline{75x^{1/3} - 150x^{1/3} + 75x^{1/3}}} + \underline{\underline{\underline{125 - 375 + 375 - 125}}}$$

$$= x$$

$$f^{-1}(f(x)) = \left[x^3 - 15x^2 + 75x - 125\right]^{1/3} + 5$$: FACTORING METHOD FOR CUBES.

$$= \left(\left[x-5\right]^3\right)^{1/3} + 5$$

$$= x - 5 + 5$$

$$= x$$

∴ FUNCTIONS MUST BE INVERSES