- **TRANSLATION OF SHAPES**

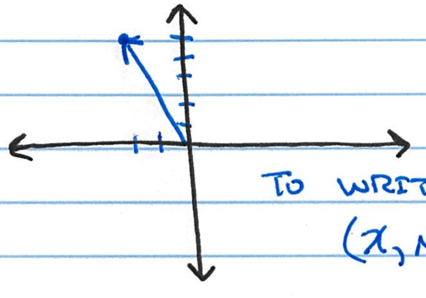
TO MOVE A SHAPE RIGHT / LEFT
AND/OR UP / DOWN

NOTATION

MAPPING $(x, y) \to (x+2, y-1)$

"MOVES RIGHT TWO, DOWN ONE"

VECTOR $\langle -2, 5 \rangle$



* THE VECTOR IS BROKEN INTO $x$ & $y$ COMPONENTS

TO WRITE IT AS A MAPPING,
$(x, y) \to (x-2, y+5)$

EXAMPLE

PRE-IMAGE → IMAGE

$(x, y) \to (x+4, y-3)$

A $(-1, 7) \to$ A' $(3, 4)$

B $(3, -2) \to$ B' $(7, -5)$

C $(-10, -4) \to$ C' $(-6, -7)$

* WHAT YOU SEE IN A MAPPING IS WHAT YOU DO.

• TRANSLATIONS OF EQUATIONS!

"SHIFTING OF PLACEMENT OF GRAPH"

3 TYPES OF GRAPHS STUDIED IN THIS SECTION.

- REVIEW OF PARABOLAS
- REVIEW OF CIRCLES
- ABSOLUTE VALUES

PARABOLAS: $y = a(x-h)^2 + K$

UNDERSTAND GRAPHING MOVEMENTS

STANDARD MOVEMENTS $|a| = 1$

$1^{ST}$ RIGHT OR LEFT 1, UP 1

$2^{nd}$ RIGHT OR LEFT 2, UP 4

$3^{RD}$ RIGHT OR LEFT 3, UP 9

ACUTE OPENINGS $|a| > 1$ {INTEGER}

$1^{ST}$ RIGHT OR LEFT 1, UP $1 \times a$

$2^{nd}$ RIGHT OR LEFT 2, UP $4 \times a$

$3^{RD}$ RIGHT OR LEFT 3, UP $9 \times a$

EXAMPLE $y = 3(x-1) + 4$

UNDO WHAT IS IN PARENTHESES → VERTEX $(1,4)$

$1^{ST}$ $\overset{1}{\leftarrow\rightarrow}$ $\uparrow 3$

$2^{nd}$ $\overset{2}{\leftarrow\rightarrow}$ $\uparrow 12$

$3^{rd}$ $\overset{3}{\leftarrow\rightarrow}$ $\uparrow 36$

OBTUSE OPENINGS $0 < |a| < 1$ PROPER FRACTION

$1^{ST}$ RIGHT OR LEFT $1 \times a$, UP $1 \times a$

$2^{nd}$ RIGHT OR LEFT $2 \times a$, UP $4 \times a$

$3^{RD}$ RIGHT OR LEFT $3 \times a$, UP $9 \times a$

EXAMPLE $y = 1/5(x+2) - 3$

VERTEX $(-2,-3)$

$1^{ST}$ $\underset{5}{\leftarrow\rightarrow}$ $\uparrow 5$

$2^{nd}$ $\underset{10}{\leftarrow\rightarrow}$ $\uparrow 20$

$3^{RD}$ $\underset{15}{\leftarrow\rightarrow}$ $\uparrow 45$

***

IF $a = (+)$ OPENS UPWARD

IF $a = (-)$ OPENS DOWNWARD

**CIRCLES:** $(x-h)^2 + (y-k)^2 = R^2$

EXAMPLE $(x+4)^2 + (y-3)^2 = 16$

UNDO WHAT IS → CENTER: $(-4, 3)$
IN PARENTHESES RADIUS: 4

FROM CENTER MOVE UP, DOWN, RIGHT, LEFT 4

**ABSOLUTE VALUE OF LINEAR FUNCTIONS:**

LIKE PARABOLAS IF $a = (+)$, OPENS UP
IF $a = (-)$, OPENS DOWN

AND VERTEX IS MAX/MIN POINT ON GRAPH

EXAMPLE 1 $y = \frac{1}{2}|x+3| - 4$

TREAT ABSOLUTE VALUE LIKE PARENTHESES "UNDO THE VALUE"

VERTEX $(-3, -4)$

GRAPHING

TO RIGHT SLOPE $= \frac{1}{2}$

TO LEFT SLOPE $= -\frac{1}{2}$

EXAMPLE 2 $y = -3|x-1| + 7$

VERTEX $(1, 7)$

GRAPHING

TO RIGHT SLOPE $= -3/1$

TO LEFT SLOPE $= 3/1$

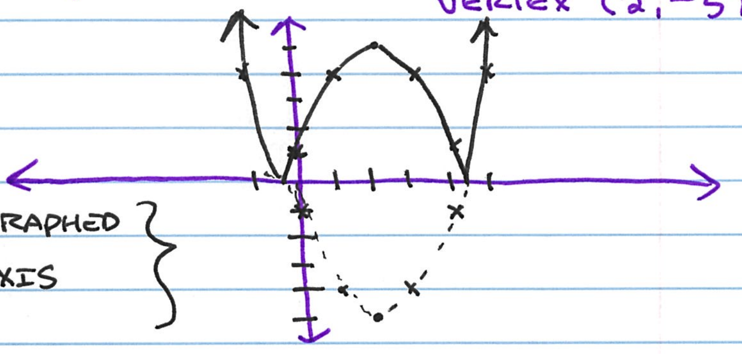
SPECIAL IDEAS ABOUT ABSOLUTE VALUE AS APPLIED TO OTHER FUNCTIONS.

- AN ABSOLUTE VALUE BY ITSELF CAN REFLECT ANY PART OF A CURVE FROM BENEATH THE X-AXIS TO ABOVE THE X-AXIS.

EXAMPLES: $y = |(x-2)^2 - 5|$

VERTEX $(2,-5)$

GRAPH

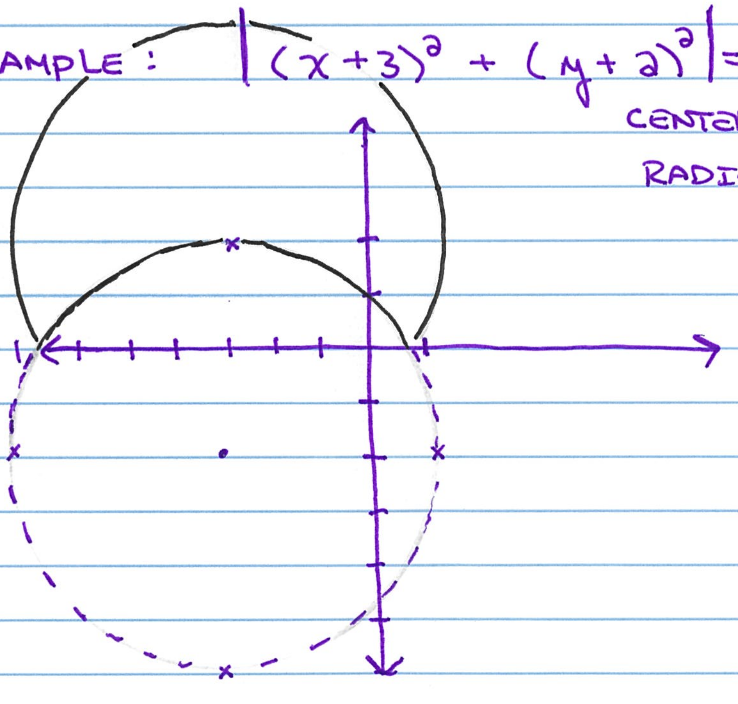


NOTHING GRAPHED BENEATH X-AXIS

EXAMPLE: $|(x+3)^2 + (y+2)^2| = 16$

CENTER $(-3,-2)$

RADIUS 4



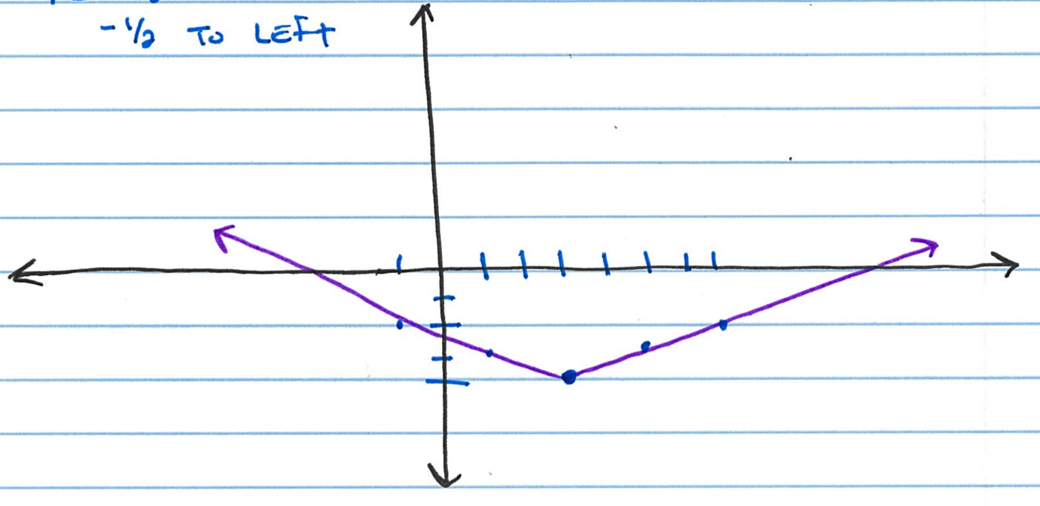
ONCE AGAIN, EVERYTHING IS REFLECTED ABOVE THE X-AXIS BECAUSE OF WHERE THE ABSOLUTE VALUE IS PLACED.

BE CAREFUL AS ABSOLUTE VALUES MAY <u>NOT</u> INFLUENCE EVERY PART OF THE CURVE. THIS IS PARTICULARLY TRUE WHEN THE ABSOLUTE VALUE DOES NOT COVER THE ENTIRE EQUATION.

EXAMPLE $y = \left|\frac{1}{2}(x-3)\right| - 4$ → SHIFTS 4 UNITS DOWN

SLOPE $\frac{1}{2}$ TO RIGHT
$-\frac{1}{2}$ TO LEFT

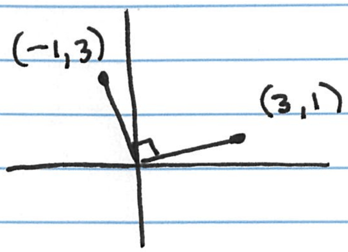
SHIFTS GRAPH 3 UNITS RIGHT



* THE NEGATIVE OUTSIDE THE ABSOLUTE VALUE ALLOWS THE GRAPH TO FALL BENEATH THE X-AXIS

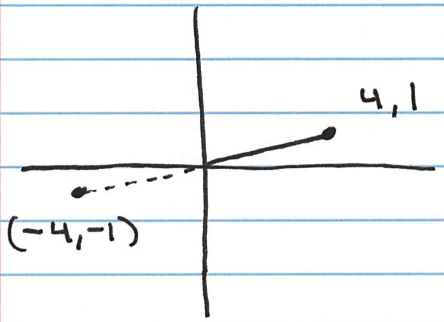
# ROTATIONS - BASIC ROTATIONS ABOUT THE ORIGIN

90° COUNTERCLOCKWISE
270° CLOCKWISE } $(x, y) \rightarrow (-y, x)$

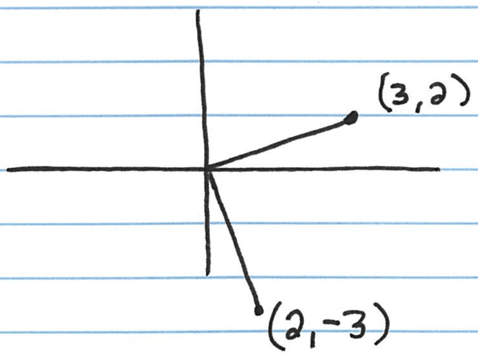



IF YOU GET CONFUSED,
DRAW A SEGMENT AND
REASON THROUGH TO
REINVENT THE MAPPING

180° ROTATION } $(x, y) \rightarrow (-x, -y)$




90° CLOCKWISE
270° COUNTERCLOCKWISE } $(x, y) \rightarrow (y, -x)$



## ROTATIONS ABOUT A POINT THAT IS NOT THE ORIGIN

I. SHIFT PRE-IMAGE WITH RESPECT TO POINT OF ROTATION

II. PERFORM ROTATION RULE

III. APPLY PIVOT POINT "SHIFT" TO RETURN TO ORIGIN PERSPECTIVE.

EXAMPLE ROTATE PRE-IMAGE 90° CLOCKWISE ABOUT (3, 4)

| GIVEN | "PHASE SHIFT" | 90° CW | SHIFT PERSPECTIVE BACK TO ORIGIN |
| --- | --- | --- | --- |
| PRE-IMAGE → | $(x-3, y-4)$ → | $(y, -x)$ → | $(x+3, y+4)$ |
| $D(-2, 3)$ → | $(-5, -1)$ → | $(-1, 5)$ → | $D'(2, 9)$ |
| $E(4, -1)$ → | $(1, -5)$ → | $(-5, -1)$ → | $E'(-2, 3)$ |
| $F(5, -3)$ → | $(2, -7)$ → | $(-7, -2)$ → | $F'(-4, 2)$ |

---

## DETERMINE THE CENTER OF ROTATION, GIVEN A PRE-IMAGE & IMAGE!

PROCESS: TAKE ANY TWO CORRESPONDING POINTS LIKE A AND A', THEN CALCULATE BOTH THE SLOPE BETWEEN A & A' AND THE MIDPOINT BETWEEN A & A'.

NEXT, WRITE THE EQUATION FOR THE LINE (⊥) PERPENDICULAR TO A & A' THROUGH THE MIDPOINT.

REPEAT THE PROCESS FOR ANOTHER SET OF POINTS LIKE B & B'. (NEXT PAGE)

THE LAST STEP OF THIS PROCEDURE WILL BE TO FIND THE POINT OF INTERSECTION FOR THE TWO EQUATIONS THAT WERE CREATED.

* THAT POINT OF INTERSECTION IS THE CENTER OF ROTATION

EXAMPLE

GIVEN $G(-2,3)$ AND $H(5,-3)$
$G'(2,9)$ $H'(-4,2)$

FIND SLOPE & MIDPOINT FOR EACH

I. $\overline{GG'}$ | $\overline{HH'}$

* MIDPOINT

(AVE x's, AVE y's)

$+4 \left\langle \begin{matrix} (-2,3) \\ (2,9) \end{matrix} \right\rangle +6$ | $-9 \left\langle \begin{matrix} (5,-3) \\ (-4,2) \end{matrix} \right\rangle +5$

MP $(0,6)$
$m = 3/2$

MP $(1/2, -1/2)$
$m = -5/9$

II. WRITE $\perp$ EQUATIONS

$m_\perp = -2/3$ THROUGH $(0,6)$
$2x + 3y = 18$

$m_\perp = 9/5$ THROUGH $(1/2, -1/2)$
$9x - 5y = 7$

III SOLVE THE SYSTEM (USING MATRIX)

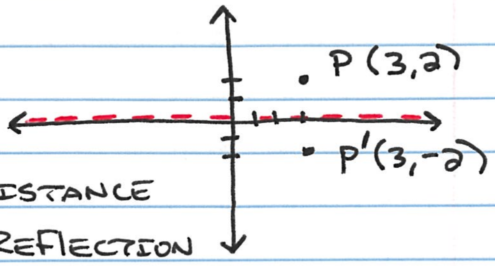
$(3,4)$

THE POINT OF INTERSECTION IS THE CENTER OF REVOLUTION

# Reflections: Basic Concept

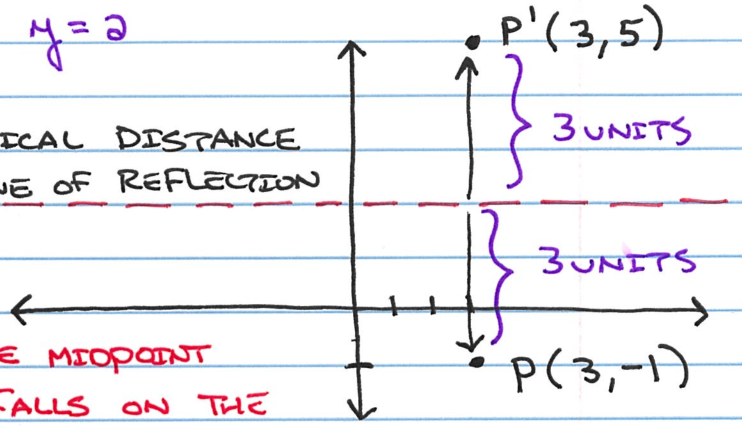
Reflect over x-axis

$$(x, y) \xrightarrow[R_{x\text{-axis}}]{} (x, -y)$$




* Count vertical distance from line of reflection

Reflect over $y = 2$




* Count vertical distance from line of reflection

** Also note the midpoint between $\overline{PP'}$ falls on the line of reflection!!!

Reflect over y-axis

$$(x, y) \xrightarrow[R_{y\text{-axis}}]{} (-x, y)$$

A $(-2, 5) \rightarrow$ A$'(2, 5)$

B $(3, -4) \rightarrow$ B$'(-3, -4)$

C $(7, 2) \rightarrow$ C$'(-7, 2)$

Reflect over line $y = x$

$$(x, y) \rightarrow (y, x)$$

D $(2, -1) \rightarrow$ D$'(-1, 2)$

E $(3, 4) \rightarrow$ E$'(4, 3)$

F $(-2, 5) \rightarrow$ F$'(5, -2)$

Graph them to see the results.

REFLECT OVER LINE $y = -x$
$(x, y) \rightarrow (-y, -x)$

$H(10, 3) \rightarrow H'(-3, -10)$
$I(-2, 4) \rightarrow I'(-4, 2)$
$J(3, -1) \rightarrow J'(1, -3)$

AGAIN, GRAPH TO VERIFY RESULTS.

## REFLECTIONS OVER ATYPICAL LINES

* OFTEN TIMES THESE REFLECTIONS RESULT IN LESS THAN IDEAL COORDINATES!!!

GIVEN POINTS $M(3, -2)$ & $N(-4, 5)$
FIND THE IMAGES $M'$ & $N'$ WHERE $M$ & $N$
WERE REFLECTED OVER $y = \frac{3}{4}x - 2$

1) FIND THE PERPENDICULAR SLOPE TO THE LINE OF REFLECTION.

$y = \frac{3}{4}x - 2 \Rightarrow m = \frac{3}{4}$ AND $m_{\perp} = -\frac{4}{3}$

2) USE PRE-IMAGE POINT TO WRITE EQUATION OF PERPENDICULAR LINE. THE SUGGESTION WOULD BE TO WRITE <u>ALL</u> EQUATIONS IN STANDARD FORM. (NOT A REQUIREMENT)

LINE OF REFLECTION IN STANDARD FORM: $3x - 4y = 8$

| | |
|---|---|
| $m_{\perp} = -4/3$ | $m_{\perp} = -4/3$ |
| $M(3, -2)$ | $N(-4, 5)$ |
| $4x + 3y = 6$ | $4x + 3y = -1$ |

3) FIND THE POINT OF INTERSECTION BETWEEN THE LINE OF REFLECTION AND THE PERPENDICULAR LINE. THIS WOULD BE THE MIDPOINT BETWEEN THE PRE-IMAGE AND IMAGE FOR ANY REFLECTED POINT.

(WORK CONTINUED)

| FOR M | FOR N |
| --- | --- |
| $3x - 4y = 8$ | $3x - 4y = 8$ |
| $4x + 3y = 6$ | $4x + 3y = -1$ |

* SOLVE THE SYSTEM, MATRIX EDITOR WITH REDUCED ROW ECHELON FORM "rref" WORKS WELL HERE.

| | |
| --- | --- |
| MP $\left(1\frac{23}{25}, -\frac{14}{25}\right)$ | MP $\left(\frac{4}{5}, -1\frac{2}{5}\right)$ |

4) APPLY THE DELTA TECHNIQUE TO MOVE FROM THE KNOWN ENDPOINT (PRE-IMAGE) TO THE MIDPOINT TO THE OTHER ENDPOINT (IMAGE).

M $(3, -2)$
$-1\frac{2}{25}$, $+1\frac{11}{25}$
MP $\left(1\frac{23}{25}, -\frac{14}{25}\right)$
$-1\frac{2}{25}$, $+1\frac{11}{25}$
M' $\left(\frac{21}{25}, \frac{22}{25}\right)$

N $(-4, 5)$
$+4\frac{4}{5}$, $-6\frac{2}{5}$
MP $\left(\frac{4}{5}, -1\frac{2}{5}\right)$
$+4\frac{4}{5}$, $-6\frac{2}{5}$
N' $\left(5\frac{3}{5}, -7\frac{4}{5}\right)$

## FINDING THE LINE OF REFLECTION.

GIVEN A AND A', WHERE A$(-5,8)$ AND A'$(7,2)$.

1) FIND THE SLOPE BETWEEN THE PAIRED POINTS

A $(-5, 8)$
$+12$, $-6$
A' $(7, 2)$

$\Rightarrow m = -\frac{1}{2}$

2) FIND THE MIDPOINT BETWEEN THE TWO POINTS AND USE THE PERPENDICULAR SLOPE ($m_\perp$) TO WRITE THE EQUATION FOR THE LINE OF REFLECTION

MP $(1, 5)$
$m_\perp = 2$

$\Bigg\}$ $y = 2x + 3$

## DILATIONS: SCALE FACTORS APPLIED TO DIMENSIONS

WHEN $K<1$, THE IMAGE WILL BE SMALLER THAN THE PRE-IMAGE.

WHEN $K=1$, THERE IS NO CHANGE IN THE PRE-IMAGE TO THE IMAGE.

WHEN $K>1$, THE IMAGE WILL BE LARGER THAN THE PRE IMAGE.

GIVEN $K=\frac{1}{2}$ CONSIDER 1 DIMENSIONAL APPLICATIONS.

IF $\triangle ABC$ HAS PERIMETER $= 15$ FT,
THEN $\triangle A'B'C'$ HAS PERIMETER $= \frac{1}{2}[15] = 7\frac{1}{2}$ FT.

IF $\triangle XYZ$ HAS ALTITUDE $= 4.3$ IN,
THEN $\triangle X'Y'Z'$ HAS ALTITUDE $= \frac{1}{2}[4.3] = 2.15$ IN.

IF $\overline{HI}$ HAS LENGTH $= 28$ cm,
THEN $\overline{H'I'} = \frac{1}{2}[28] = 14$ cm.

* FOR 1 DIMENSION K IS ONLY APPLIED ONCE.

GIVEN $K=2$ CONSIDER 2 DIMENSIONAL APPLICATION.

IF $\triangle DEF$ HAS AREA $= 14$ FT$^2$,
THEN $\triangle D'E'F' = [2]^2 \cdot 14 = 56$ FT$^2$.

GIVEN $K=\frac{1}{3}$ CONSIDER 2 DIMENSIONAL APPLICATION.

IF TRAPEZOID ABCD HAS AREA $= 40$ cm$^2$,
THEN TRAPEZOID $A'B'C'D = [\frac{1}{3}]^2 \cdot 40 = \frac{40}{9}$ cm$^2$

* FOR 2 DIMENSIONS K IS APPLIED TWICE, "$K^2$" MATCHES THE UNITS.

GIVEN $K=\frac{1}{2}$ CONSIDER 3 DIMENSIONAL APPLICATION.

IF PRISM $l \cdot w \cdot k$ HAS VOLUME $= 240$ FT$^3$,
THEN PRISM $l' \cdot w' \cdot k' = [\frac{1}{2}]^3 \cdot 240 = 30$ FT$^3$.

GIVEN $K=3$ CONSIDER 3 DIMENSIONAL APPLICATION.

IF PRISM $l \cdot w \cdot k$ HAS VOLUME $= 5$ cm$^3$,
THEN PRISM $l' \cdot w' \cdot k' = [3]^3 \cdot 5 = 135$ cm$^3$

* FOR 3 DIMENSIONS K IS APPLIED 3 TIME $= K^3$.

## DILATIONS ABOUT THE ORIGIN:

GIVEN $\Delta ABC$ WITH $A(-1,3)$, $B(2,-4)$, $C(5,2)$ AND SCALE FACTOR $K=4$ FIND COORDINATES OF $A'$, $B'$, AND $C'$.

SINCE EVERYTHING IS SCALED BY THE ORIGIN

$$(x,y) \rightarrow (K \cdot x, K \cdot y)$$

$A(-1,3) \rightarrow A'(-4,12)$

$B(2,-4) \rightarrow B'(8,-16)$

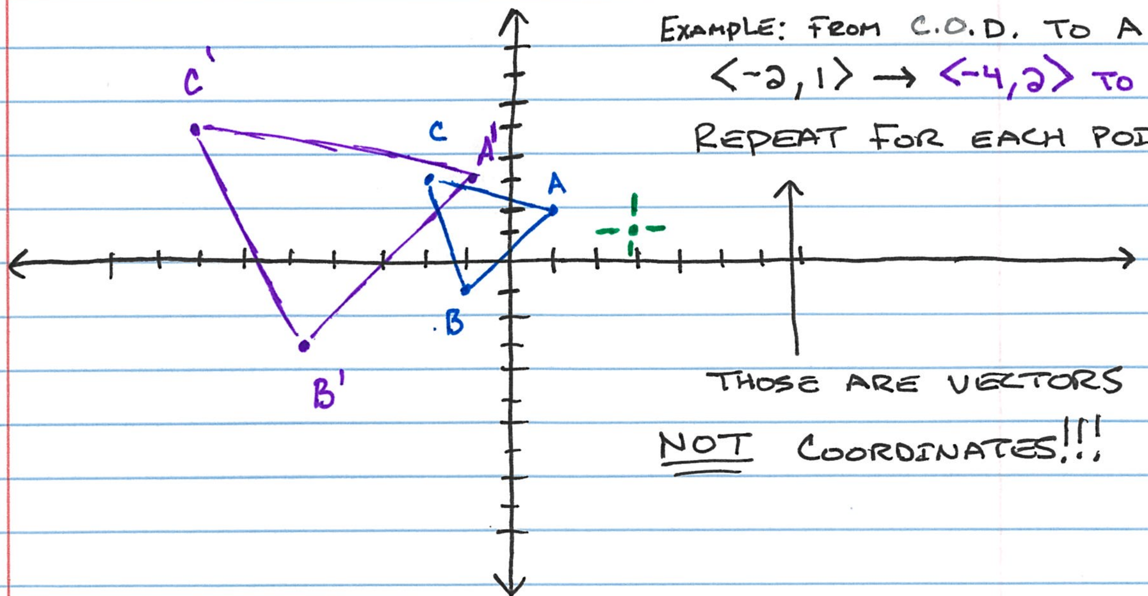
$C(5,2) \rightarrow C'(20,8)$

## DILATIONS NOT CENTERED AT THE ORIGIN:

CONSIDER THIS GEOMETRIC OR GRAPHING APPROACH

WITH $A(1,2)$, $B(-1,-1)$, AND $C(-2,3)$ DILATE $\Delta ABC$ WITH CENTER OF DILATION $(3,1)$ AND SCALE FACTOR OF $K=2$

1) PLOT THE CENTER OF DILATION
2) DETERMINE THE VECTOR LENGTHS FROM THE CENTER OF DILATION TO EACH PRE-IMAGE
3) MULTIPLY EACH LENGTH BY THE SCALE FACTOR AND
4) PLOT THE IMAGE AS ONE COUNTS FROM THE CENTER OF DILATION.




EXAMPLE: FROM C.O.D. TO A

$\langle -2, 1\rangle \rightarrow \langle -4, 2\rangle$ TO $A'$

REPEAT FOR EACH POINT.

THOSE ARE VECTORS NOT COORDINATES!!!

## DILATIONS NOT CENTERED AT THE ORIGIN:

CONSIDER THE **ALGEBRAIC APPROACH** TO CREATE A MAPPING OR RULE

* WE DESCRIBED A FORMULA FOR ANY GENERAL POINT OR SCENARIO AS FOLLOWS.

$$P(x,y) \rightarrow P'\left(k(x-D_x)+D_x,\ k(y-D_y)+D_y\right)$$

WHERE $K$ IS THE SCALE FACTOR AND THE CENTER OF DILATION IS $(D_x, D_y)$

THE RULE WILL ALWAYS NEED TO BE ALGEBRAICALLY SIMPLIFIED.

EXAMPLE: CONSIDER $D(-2,3)$, $E(4,-1)$, AND $F(3,4)$

USE CENTER OF DILATION $(6,-2)$ AND SCALE FACTOR $K=3$

$$P(x,y) \rightarrow P'(3(x-6)+6,\ 3(y+2)-2)$$

$$\rightarrow P'(3x-12,\ 3y+4)$$

$$(x,y) \rightarrow (3x-12,\ 3y+4)$$ "RULE"

$$\left.\begin{array}{l} D(-2,3) \rightarrow D'(-18,13) \\ E(4,-1) \rightarrow E'(0,1) \\ F(3,4) \rightarrow F'(-3,16) \end{array}\right\}$$ PLUG IN VALUES FOR $x$ AND $y$ EXPRESSIONS RESPECTIVELY

## FINDING CENTERS OF DILATION:

ANY LINE DEFINED BY $P$ & $P'$ WILL CONTAIN THE CENTER OF DILATION. SO ANY INTERSECTION BETWEEN TWO SETS OF LINES WILL PRODUCE THE CENTER OF DILATION.

(NEXT PAGE FOR EXAMPLE)

EXAMPLE: CONSIDER THE PREVIOUS EXAMPLE

$-16 \left\langle \begin{array}{l} D\ (-2,3) \\ D'\ (-18,13) \end{array} \right\rangle +10$ $\qquad$ $-4 \left\langle \begin{array}{l} E\ (4,-1) \\ E'\ (0,1) \end{array} \right\rangle +2$

$m = \frac{10}{-16} = \frac{-5}{8}$ $\qquad$ $m = \frac{2}{-4} = -\frac{1}{2}$

- WRITE STANDARD FORM EQUATIONS

$5x + 8y =$ $\qquad$ $x + 2y =$

$5[-2] + 8[3] =$ $\qquad$ $[0] + 2[1] =$

$-10 + 24 =$ $\qquad$ $0 + 2 =$

$14 =$ $\qquad$ $2 =$

$5x + 8y = 14$ $\qquad$ $x + 2y = 2$

- PUT INTO MATRIX & FIND POINT OF INTERSECTION

$$\begin{bmatrix} 5x + 8y = 14 \\ x + 2y = 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -2 \end{bmatrix}$$

WHICH LEADS TO THE CENTER OF DILATION

$(6, -2)$

OBVIOUSLY THIS WAS THE EXPECTED RESULT SINCE THE PREVIOUS EXAMPLE WAS RECYCLED.