Introduction To Geometry

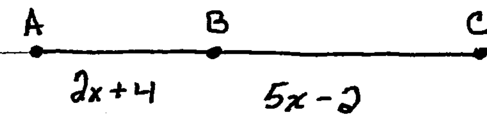
Some key concepts to remember

Segment Addition

If $\overline{AC} = 30$

$\overline{AB} = 2x+4$

$\overline{BC} = 5x-2$



Then an equation can be written where

$$\overline{AB} + \overline{BC} = \overline{AC}$$

$\overline{AB} = 2x+4$	$\overline{BC} = 5x-2$
$= 2(4)+4$	$= 5(4)-2$
$= 12$	$= 18$

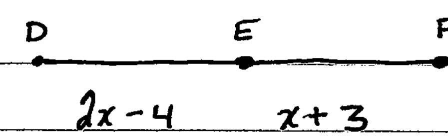
$$2x+4+5x-2 = 30$$
$$7x+2 = 30$$
$$7x = 30-2$$
$$7x = 28$$
$$x = 4$$

* There is more to these problems than solving an equation. The objective is to find the length of each segment.

Midpoint To Segment

If a midpoint is given then it will divide the segment into two equal parts.

If DF has a midpoint at E and $\overline{DE} = 2x-4$, $\overline{EF} = x+3$



Then the segments can be set equal to one another

$$\overline{DE} = \overline{EF}$$

$\overline{DE} = 2x-4$	$\overline{EF} = x+3$
$= 2(7)-4$	$= 7+3$
$= 10$	$= 10$

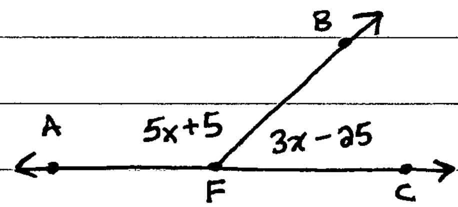
$$2x-4 = x+3$$
$$2x-x = 3+4$$
$$x = 7$$

ANGLES – THE ARCH MEASURE BETWEEN INTERSECTING LINES OR RAYS.

LINEAR PAIR – WHEN TWO ANGLES ARE DRAW IN SUCH A WAY THAT THEY HELP TO FORM A LINE THEY FORM THIS **SUPPLEMENTARY** RELATIONSHIP.

SUPPLEMENTARY ANGLES – THE SUM OF THE ANGLES WILL BE $180°$

EXAMPLE:



$m\angle AFB + m\angle BFC = m\angle AFC$: $m\angle AFC$ IS LINEAR $= 180°$

$5x+5 + 3x-25 = 180$

$8x - 20 = 180$

$8x = 180 + 20$

$\frac{8x}{8} = \frac{200}{8}$

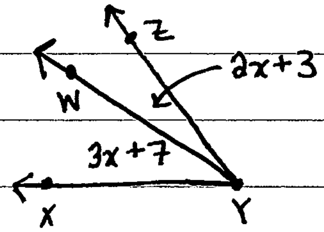
$x = 25$

$m\angle AFB = 5x+5$
$= 5(25)+5$
$= 130°$

$m\angle BFC = 3x-25$
$= 3(25)-25$
$= 50°$

ANGLE ADDITION

EXAMPLE $m\angle XYZ = 70$



$m\angle XYW + m\angle WYZ = m\angle XYZ$

$3x+7 + 2x+3 = 70$

$5x + 10 = 70$

$5x = 70 - 10$

$\frac{5x}{5} = \frac{60}{5}$

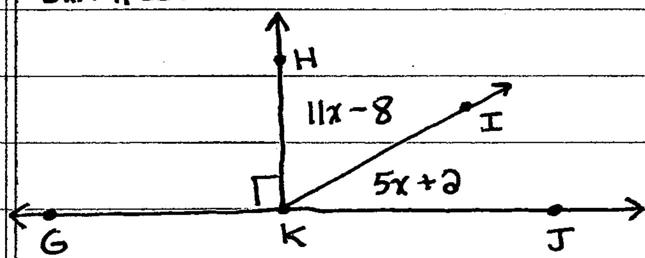
$x = 12$

$m\angle XYW = 3x+7$
$= 3(12)+7$
$= 36+7$
$= 43°$

$m\angle WYZ = 2x+3$
$= 2(12)+3$
$= 24+3$
$= 27°$

COMPLEMENTARY ANGLES - WHEN ANGLES ARE COMBINED TO FORM A RIGHT ANGLE, THE SUM WILL EQUAL 90°

EXAMPLE:



$m\angle IKJ + m\angle HKI = m\angle HKJ$

$5x+2 + 11x-8 = 90$

$16x-6 = 90$

$16x = 90+6$

$\frac{16x}{16} = \frac{96}{16}$

$x = 6$

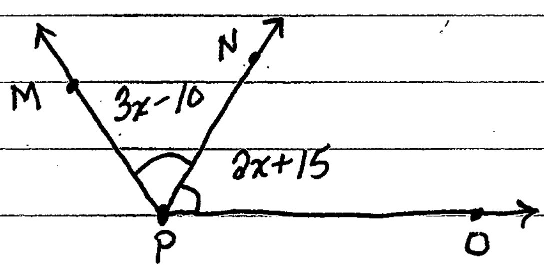
So $m\angle IKJ = 5x+2$ AND $m\angle HKI$

$= 5(6)+2$ $\quad$ $= 90-32$

$= 32^\circ$ $\quad$ $= 58^\circ$

* INSTEAD OF PLUGGING $x=6$ BACK INTO EACH EXPRESSION YOU COULD FIND THE ANSWER FOR ONE OF THE EXPRESSIONS THEN USE THE COMPLEMENTARY PROPERTY TO FIND THE OTHER

ANGLE BISECTOR - MUCH AS A MIDPOINT DIVIDES A SEGMENT INTO TWO EQUAL PARTS, THE ANGLE BISECTOR WILL DIVIDE THE ANGLE INTO TWO ANGLES OF EQUAL MEASURE.



IF $\overrightarrow{NP}$ BISECTS $\angle MPO$

THEN $m\angle MPN = m\angle NPO$

$3x-10 = 2x+15$

$3x-2x = 15+10$

$x = 25$

So $m\angle MPN = m\angle NPO = 2x+15$

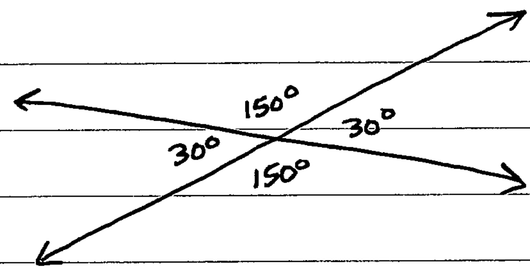
$= 2(25)+15$

$= 65^\circ$

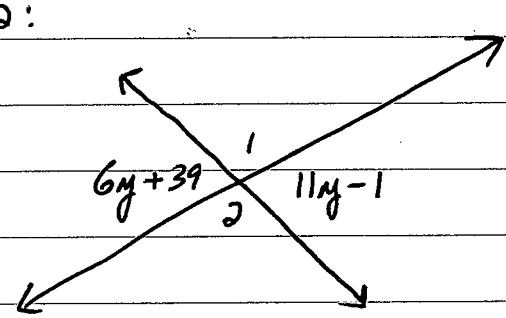
ADDITIONALLY ONE COULD FIGURE $m\angle MPO = 130^\circ$

VERTICAL ANGLES - WHEN TWO LINES INTERSECT THE RESULT WILL BE TWO SETS OF ANGLES WITH EQUAL MEASURE. IN OTHER WORDS, THE NONSUPPLEMENTAL ANGLES WILL BE EQUAL.

EXAMPLE 1:



EXAMPLE 2:



$6y + 39 = 11y - 1$

$39 + 1 = 11y - 6y$

$40 = 5y$

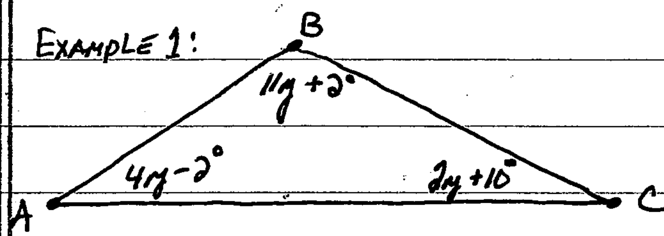
$8 = y$

$$\boxed{\begin{array}{l} \text{So } 11(8) - 1 = 87^\circ \\ \text{AND } m\angle 1 = 180 - 87 = 93^\circ \end{array}}$$

TRIANGLES

ANGLE SUM FOR TRIANGLE - THE SUM OF THE ANGLES (INSIDE) THE TRIANGLE MUST BE 180°

EXAMPLE 1:



$m\angle A + m\angle B + m\angle C = 180$

$4y-2 + 11y+2 + 2y+10 = 180$

$17y + 10 = 180$

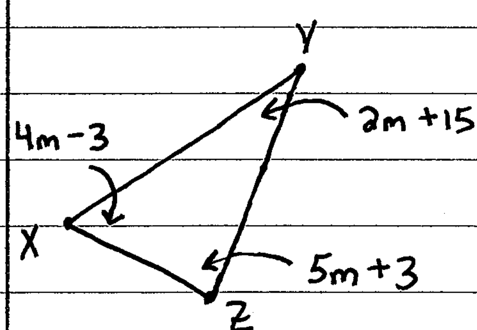
$17y = 180 - 10$

$17y = 170$

$y = 10$

$$\boxed{\begin{array}{l} m\angle A = 4y-2 = 38^\circ \\ m\angle B = 11y+2 = 112^\circ \\ m\angle C = 2y+10 = 30^\circ \end{array}}$$

EXAMPLE 2



$m\angle X + m\angle Y + m\angle Z = 180$

$4m-3 + 2m+15 + 5m+3 = 180$

$11m + 15 = 180$

$11m = 180 - 15$

$11m = 165$

$m = 15$

$m\angle X = 4m-3 = 4(15)-3 = 57^\circ$

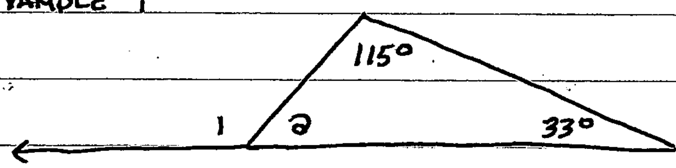
$m\angle Y = 2m+15 = 2(15)+15 = 45^\circ$

$m\angle Z = 5m+3 = 5(15)+3 = 78^\circ$

EXTERIOR ANGLE THEOREM

– THE ANGLE EXTERIOR AND ADJACENT TO AN INTERIOR ANGLE WILL FORM A SUPPLEMENTARY RELATIONSHIP, AND THE EXTERIOR ANGLE WILL BE EQUAL TO THE SUM OF THE TWO NON-ADJACENT INTERIOR ANGLES.

EXAMPLE 1



$m\angle 1 + m\angle 2 = 180$

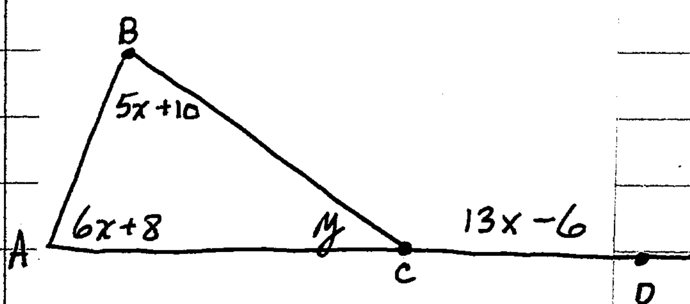
$m\angle 1 = 180 - m\angle 2$

$m\angle 1 = 115 + 33$

$\boxed{m\angle 1 = 148°}$

AND SO $m\angle 2 = 32°$

EXAMPLE 2



$m\angle BCD = m\angle ABC + m\angle CAB$

$13x - 6 = 5x + 10 + 6x + 8$

$13x - 6 = 11x + 18$

$13x - 11x = 18 + 6$

$2x = 24$

$x = 12$

$m\angle ABC = 5x + 10 = 5(12) + 10 = 70°$

$m\angle BAC = 6x + 8 = 6(12) + 8 = 80°$

$m\angle BCD = 13x - 6 = 13(12) - 6 = 150°$

$m\angle y = 30°$

MIDPOINT FORMULA

* SOMETIMES THE MIDPOINT WILL BE DISCUSSED AS A POINT ON THE COORDINATE PLANE.

EXAMPLE: FIND THE MIDPOINT OF $\overline{AC}$ IF $A(-3,7)$ AND $C(4,-3)$ ARE THE ENDPOINTS

SOLUTION $\left.\begin{array}{l} A(-3,7) \\ C(4,-3) \end{array}\right\}$ ADD THE x-COORDINATE VALUES AND DIVIDE BY 2, THEN DO THE SAME FOR THE y-COORDINATES.

x: $\frac{-3+4}{2} = \frac{1}{2}$

y: $\frac{7+(-3)}{2} = \frac{4}{2} = 2$ $\boxed{\text{MIDPOINT TO } \overline{AC} = (\tfrac{1}{2}, 2)}$

EXAMPLE 2: FIND THE MIDPOINT E TO $\overline{DF}$ IF $D(11,3)$ AND $F(5,-7)$ ARE ENDPOINTS

SOLUTION $D(11,3)$
$F(5,-7)$

$$\left.\begin{array}{l} x: \frac{11+5}{2} = \frac{16}{2} = 8 \\ y: \frac{3+-7}{2} = \frac{-4}{2} = -2 \end{array}\right\}$$

$\boxed{\text{SO MIDPOINT TO } \overline{DF} \text{ CALLED } E(8,-2)}$

A VARIATION TO THIS PROBLEM OF MIDPOINTS IS WHEN ONE NEEDS TO FIND AN ENDPOINT GIVEN THE MIDPOINT AND ONE OF THE OTHER ENDPOINTS.

EXAMPLE: FIND X IF THE MIDPOINT IS $Y(5,-3)$ AND Z THE OTHER ENDPOINT TO $\overline{XZ}$ IS $Z(-2,-5)$.

SOLUTION $X(x,y)$ MIDPOINT $Y(5,-3)$
$Z(-2,-5)$

SO $\frac{x+-2}{2}=5$ AND $\frac{y+(-5)}{2}=-3$

$\frac{x-2}{2}=5$ $\quad$ $\frac{y-5}{2}=-3$ $\quad$ $\boxed{X(12,-1)}$

$x-2=10$ $\quad$ $y-5=-6$

$x=12$ $\quad$ $y=-1$

EXAMPLE 2: MIDPOINT TO $\overline{AC}$ IS FOUND AT $B(1,-5)$ ENDPOINT $A(3,2)$ FIND COORDINATES FOR C.

SOLUTION $A(3,2)$ MIDPOINT $(1,-5)$
$C(x,y)$

SO $\frac{x+3}{2}=1$ AND $\frac{y+2}{2}=-5$ $\quad$ $\boxed{C(-1,-12)}$

$x+3=2$ $\quad$ $y+2=-10$

$x=2-3$ $\quad$ $y=-10-2$

$x=-1$ $\quad$ $y=-12$