Calculus (Take Home) Test 1 **Name**_____

1) Write the equation of each circle

a. (1 , –1)
(4 , –2)
(0 , –4)

b. (3 , –2)
(1 , 4)
(6 , 7)

a $(x-2)^2 + (y+3)^2 = 5$

b $(x-5\frac{1}{2})^2 + (y-2\frac{1}{6})^2 = 23\frac{11}{18}$ **will be nasty!**

2) Find the equation of x and y such that $\sqrt{2}$ times the distance between (x , y) and (2 , –3) is $\sqrt{3}$ times the distance between (x , y) and (–3 , 4).

2 $(x+13)^2 + (y-18)^2 = 444$

3) Explain two different ways to determine if A(– 4 , 1), B(3 , 9), and C(5, –2) are collinear .

SEE ENCLOSED WORK
[5 METHODS]

4) Find the vertices of a triangle if it has midpoints at (-3 , 5), (2, 3), (7, 4).

4 (2,6), (-8,4), (12,2)

5) Solve $5|3x+5|^2 - 21|3x+5| = -20$

5 $x = \{-3, -1\frac{11}{15}, -1\frac{3}{5}, -\frac{1}{3}\}$

6) Solve the following systems of equations

a. $\begin{aligned} 2x-3y+5z &= 13 \\ 6x+3y-2z &= 7 \\ 2x+5y+3z &= 29 \end{aligned}$

b. $\begin{aligned} 3x+5y &= 25 \\ 4x-y &= -5 \end{aligned}$

c. $\begin{aligned} 3x-2y+z &= 4 \\ x+4y-z &= 11 \\ x-2y+2z &= 6 \end{aligned}$

a $(1, 3, 4)$ b $(0, 5)$ c $(1\frac{8}{9}, 3\frac{13}{18}, 5\frac{7}{9})$

d. $\begin{aligned} 4x+2y-6z &= 9 \\ x-5y+4z &= 6 \\ 2x+y+3z &= -3 \end{aligned}$

e. $\begin{aligned} x+3y-2z &= 4 \\ 4x-y+z &= -1 \\ -2x-6y+4z &= -8 \end{aligned}$

d $(1\frac{15}{44}, -1\frac{41}{44}, -1\frac{1}{4})$ e INFINITE SOLUTIONS

7) If $f(x) = 3x-2$, $g(x) = x^2-2x-8$, $h(x) = x^2+2$

a. find $f(1) + g(4) + h(0)$ a 3

b. find $f(f(2))$ b 10

c. find $(g(h(f(0)))$ c 16

d. find the **solution set** for $f(g(x)) = g(f(x)$
five decimal places if appropriate

d NON REAL SOLUTIONS

#1A $x^2 + y^2 + Ax + By + C = 0$

$(1,-1)$ $\quad 1 + 1 + A - B + C = 0$

$A - B + C = -2$

$(4,-2)$ $\quad 16 + 4 + 4A - 2B + C = 0$

$4A - 2B + C = -20$

$(0,-4)$ $\quad 0 + 16 + 0A - 4B + C = 0$

$0A - 4B + C = -16$

USE RREF ON 3x4 MATRIX

$A = -4 \quad B = 6 \quad C = 8$

$x^2 - 4x + \underline{4} + y^2 + 6y + \underline{9} + 8 = 0 + \underline{13}$

$(x-2)^2 + (y+3)^2 = 13 - 8$

$(x-2)^2 + (y+3)^2 = 5$

#1B $(3,-2)$ $\quad x^2 + y^2 + Ax + By + C = 0$

$9 + 4 + 3A - 2B + C = 0$

$3A - 2B + C = -13$

$(1,4)$ $\quad 1 + 16 + A + 4B + C = 0$

$A + 4B + C = -17$

$(6,7)$ $\quad 36 + 49 + 6A + 7B + C = 0$

$6A + 7B + C = -85$

USE RREF ON 3x4 MATRIX

$A = -11 \quad B = -\frac{13}{3} \quad C = \frac{34}{3}$

$x^2 - 11x + \frac{121}{4} + y^2 - \frac{13}{3}y + \frac{169}{36} = -\frac{34}{3} + \frac{121}{4} + \frac{169}{36}$

$\left(x - \frac{11}{2}\right)^2 + \left(y - \frac{13}{6}\right)^2 = \frac{425}{18}$

$(x - 5\frac{1}{2})^2 + (y - 2\frac{1}{6})^2 = 23\frac{11}{18}$

#2

$$\sqrt{2}\cdot\sqrt{(x-2)^2+(y+3)^2} = \sqrt{3}\cdot\sqrt{(x+3)^2+(y-4)^2} \quad : \text{SQUARE BOTH SIDES}$$

$$2\left[(x-2)^2+(y+3)^2\right] = 3\left[(x+3)^2+(y-4)^2\right]$$

$$2\left[x^2-4x+4+y^2+6y+9\right] = 3\left[x^2+6x+9+y^2-8y+16\right]$$

$$2x^2-8x+2y^2+12y+26 = 3x^2+18x+3y^2-24y+75$$

$$0 = x^2+26x+y^2-36y+49$$

$$-49+169+324 = x^2+26x+\underline{169}+y^2-36y+\underline{324}$$

$$444 = (x+13)^2+(y-18)^2$$

#3

A] AREA of TRIANGLE (COLLINEAR POINTS WOULD HAVE NO AREA)

$$\frac{1}{2}\det\begin{vmatrix} -4 & 1 & 1 \\ 3 & 9 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \frac{93}{2}$$

$\therefore$ POINTS ARE <u>NOT</u> COLLINEAR

B] DISTANCES

DISTANCE AC $(-4,1)$, $(5,-2)$

$$d = \sqrt{\Delta x^2+\Delta y^2} = \sqrt{9^2+3^2} = \sqrt{90}$$

DISTANCE AB $(-4,1)$, $(3,9)$

$$d = \sqrt{\Delta x^2+\Delta y^2} = \sqrt{7^2+8^2} = \sqrt{113}$$

DISTANCE BC $(3,9)$, $(5,-2)$

$$d = \sqrt{\Delta x^2+\Delta y^2} = \sqrt{2^2+11^2} = \sqrt{125}$$

$\sqrt{125} \neq \sqrt{90}+\sqrt{113}$

$\therefore$ POINTS ARE <u>NOT</u> COLLINEAR

C] SLOPES

SLOPE AC $9 < (-4,1), (5,-2) > -3$ $\quad m = -1/3$

SLOPE AB $+7 < (-4,1), (3,9) > +8$ $\quad m = 8/7$

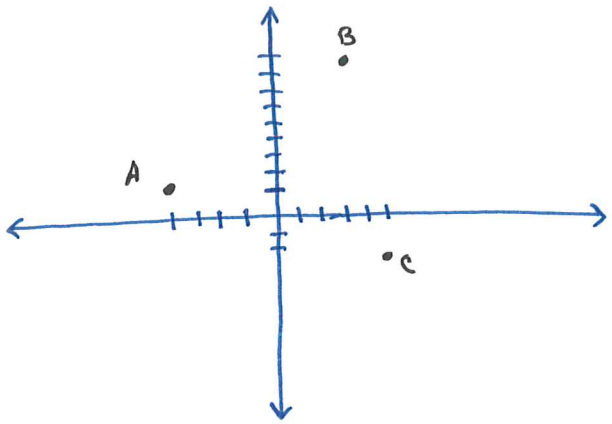
SLOPE BC $+2 < (3,9), (5,-2) > -11$ $\quad m = -11/2$

SLOPES ARE NOT EQUAL

$\therefore$ POINTS ARE NOT COLLINEAR

#3

D] GRAPH THE POINTS (COULD BE CRUDE IF POINTS ARE VERY CLOSE)



CLEARLY THE POINTS DO NOT FORM A LINE

E] CREATE THE EQUATION OF A LINE BETWEEN TWO POINTS THEN CHECK THE THIRD AS MEMBER OF SOLUTION SET.

A (-4, 1)
B (3, 9)
$m = 8/7$

$8x - 7y = C$
$8(3) - 7(9) = C$
$-39 = C$

$8x - 7y = -39$

NEXT CHECK C AS MEMBER OF SOLUTION SET

C (5, -2)

$8(5) - 7(-2) \stackrel{?}{=} -39$
$40 + 14 \stackrel{?}{=} -39$
$54 \neq -39$

$\therefore$ POINTS ARE <u>NOT</u> COLLINEAR.

#4 Find Vertices of Triangle Given Midpoints

Let MP to AB = $(-3, 5)$

BC = $(2, 3)$

AC = $(7, 4)$

For x-coordinates

$$\frac{A+B}{2} = -3$$

$$\frac{B+C}{2} = 2$$

$$\frac{A+C}{2} = 7$$

$$\begin{bmatrix} A + B = -6 \\ B + C = 4 \\ A + C = 14 \end{bmatrix}$$

* RREF of matrices

$A = 2 \quad B = -8 \quad C = 12$

For y-coordinates

$$\frac{A+B}{2} = 5$$

$$\frac{B+C}{2} = 3$$

$$\frac{A+C}{2} = 4$$

$$\begin{bmatrix} A + B = 10 \\ B + C = 6 \\ A + C = 8 \end{bmatrix}$$

$A = 6 \quad B = 4 \quad C = 2$

Vertices

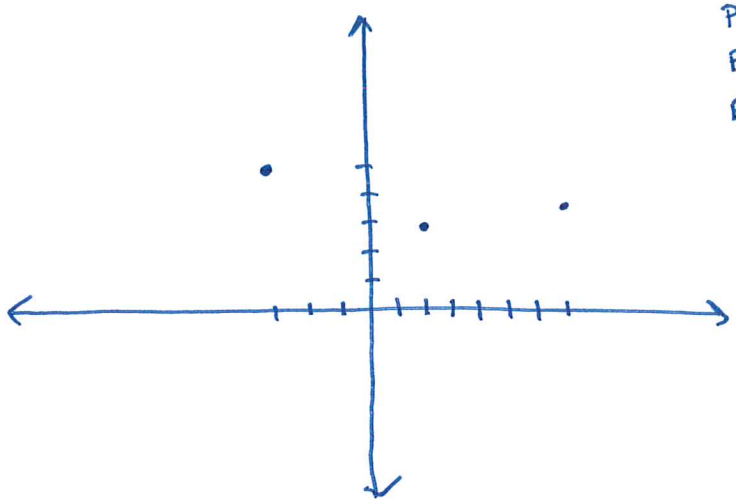
A $(2, 6)$

B $(-8, 4)$

C $(12, 2)$

* Labels are arbitrary

#4 GRAPHING APPROACH



POINT 1: (-3,5)
POINT 2: (2,3)
POINT 3: (7,4)

FIRST CONSIDER SLOPE BETWEEN POINT 1 AND POINT 2

$5 < \begin{matrix}(-3,5)\\(2,3)\end{matrix} > -2 \qquad m = -2/5$

WHEN THAT SLOPE IS APPLIED TO POINT 3 ONE WILL LOCATE TWO OF THE VERTICES.

$+5 < (2,6) > -2$

$+5 < \begin{matrix}(7,4)\\(12,2)\end{matrix} > -2$

TWO VERTICES
(2,6)
(12,2)

SECOND FIND SLOPE BETWEEN POINT 2 AND POINT 3

$+5 < \begin{matrix}(2,3)\\(7,4)\end{matrix} > +1 \qquad m = 1/5$

APPLY THAT SLOPE TO POINT 1

$5 < \begin{matrix}(-8,4)\\(-3,5)\end{matrix} > 1$

$5 < (2,6) > 1$

ALREADY HAD (2,6) BUT FOUND THIRD VERTICE (-8,4)

VERTICES
(2,6)
(-8,4)
(12,2)

#5 Solve $5|3x+5|^2 - 21|3x+5| = -4$

Let $E = |3x+5|$

$5E^2 - 21E + 4 = 0$

AC = 20 B = -21

-20 -1

$(5E-1)(E-4) = 0$

$E = 1/5 \quad E = 4$

$|3x+5| = 1/5 \quad |3x+5| = 4$

$3x+5 = 1/5 \quad 3x+5 = -1/5 \quad 3x+5 = 4 \quad 3x+5 = -4$

$\frac{3x}{3} = \frac{-4\,4/5}{3} \quad \frac{3x}{3} = \frac{-5\,1/5}{3} \quad \frac{3x}{3} = \frac{-1}{3} \quad \frac{3x}{3} = \frac{-9}{3}$

$x = -1\,3/5 \quad x = -1\,11/15 \quad x = -1/3 \quad x = -3$

$x = \{-3, -1\,11/15, -1\,3/5, -1/3\}$

#6 ENTER EACH SYSTEM AS MATRIX THEN USE RREF OPTION

A] (1, 3, 4) B] (0, 5) C] $(1\,8/9, 3\,13/18, 5\,7/9)$

$(17/9, 67/18, 52/9)$

D] $(1\,15/44, -1\,41/44, -1\,1/4)$

$(59/44, -85/44, -5/4)$

E]
$$\begin{bmatrix} 1 & 0 & 1/13 & 1/13 \\ 0 & 1 & -9/13 & 17/13 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

PARAMETRIC

(INFINITE SOLUTIONS)

#7 $f(x) = 3x-2$ $g(x) = x^2-2x-8$ $h(x) = x^2+2$

A] $f(1) = 3[1]-2$
$= 3-2$
$= 1$
$g(4) = [4]^2-2[4]-8$
$= 16-8-8$
$= 0$
$h(0) = [0]^2+2$
$= 2$

$f(1)+g(4)+h(0)$
$1 + 0 + 2$
$\boxed{= 3}$

B] $f(f(2))$
$f(4)$
$\boxed{= 10}$

$f(2) = 3[2]-2$
$= 6-2$
$= 4$
$f(4) = 3[4]-2$
$= 12-2$
$= 10$

C] $g(h(f(0)))$
$g(h(-2))$
$g(6)$
$\boxed{= 16}$

$f(0) = 3[0]-2$
$= -2$
$h(-2) = [-2]^2+2$
$= 4+2$
$= 6$
$g(6) = [6]^2-2[6]-8$
$= 36-12-8$
$= 16$

D] $f(g(x)) = g(f(x))$

L.H.S. $= f(g(x))$
$= 3[x^2-2x-8]-2$
$= 3x^2-6x-24-2$
$= 3x^2-6x-26$

R.H.S. $= g(f(x))$
$= [3x-2]^2-2[3x-2]-8$
$= 9x^2-12x+4-6x+4-8$
$= 9x^2-18x-8+8$
$= 9x^2-18x$

$3x^2-6x-26 = 9x^2-18x$
$0 = 6x^2-12x+26$

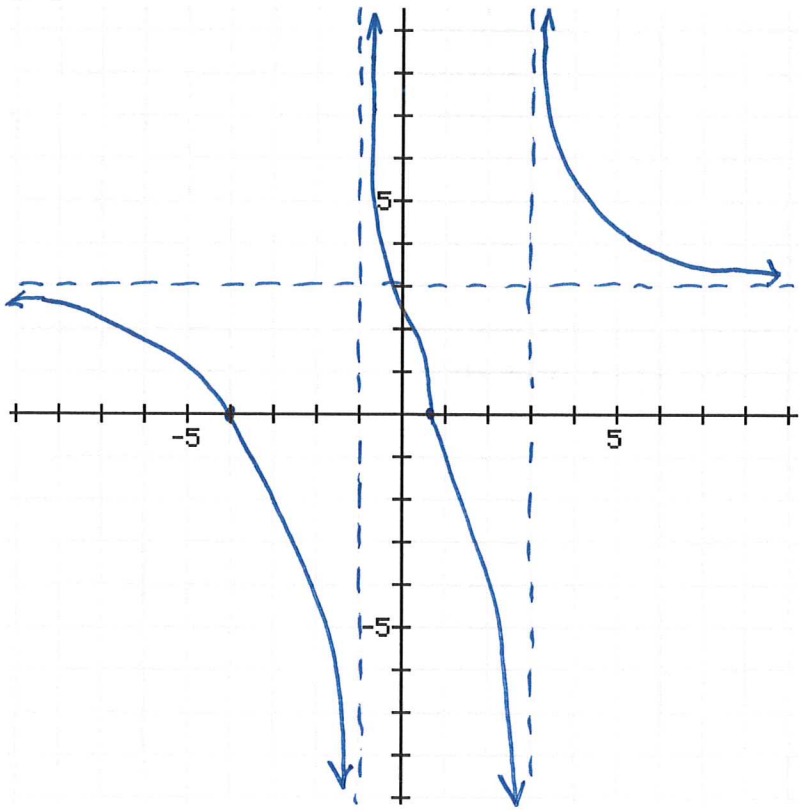
NON REAL SOLUTION

Graph and identify zeroes, all asypmtotes, and perform a sign check, remember to check extreme values.

$AC = -24 \quad B = 10$
$12 \quad -2$

9. $y = \dfrac{3x^2 + 10x - 8}{x^2 - 2x - 3}$ $\dfrac{(3x-2)(x+4)}{(x-3)(x+1)}$

graph



zeroes $x = \{-4, 2/3\}$

vertical asymptotes $x \neq \{-1, 3\}$

horizontal asymptotes $y = 3$

sign check (+) 0 (−) DNE (+) 0 (−) DNE (+)

$\lim_{x \to -\infty} f(x) = 3^-$ ← − − − −4 − − − −1 − − − 2/3 − − − 3 − − − → $\lim_{x \to \infty} f(x) = 3^+$

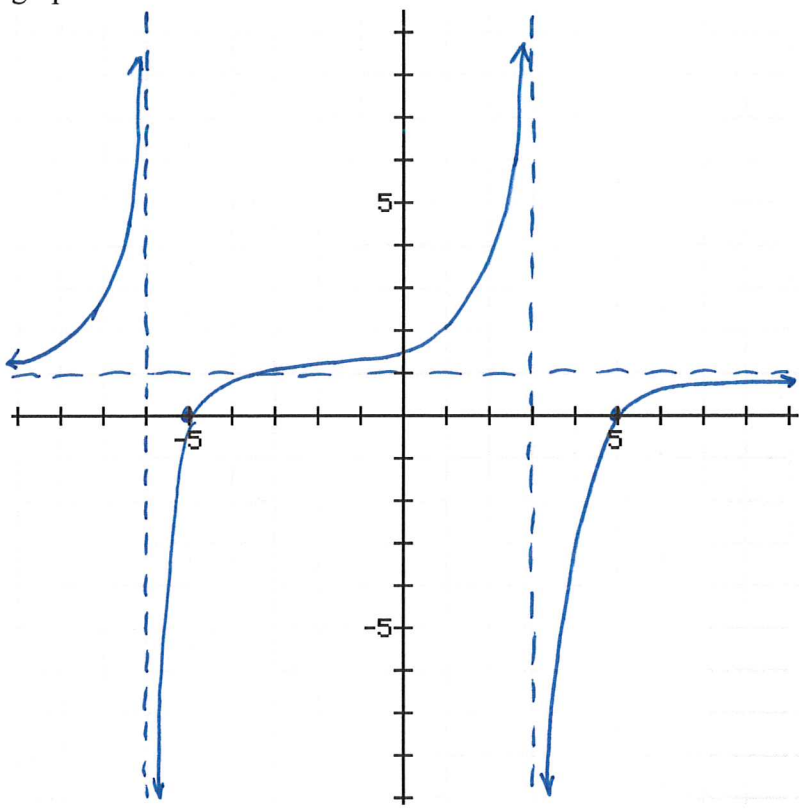
State the domain and range using **interval notation.**

Domain: $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$ Range: $(-\infty, \infty)$

Graph and identify zeroes, all asypmtotes, and perform a sign check.

10. $\dfrac{x^2-25}{x^2+3x-18}$ $\dfrac{(x-5)(x+5)}{(x+6)(x-3)}$

graph



zeroes $x = \{-5, 5\}$

vertical asymptotes $x \neq \{-6, 3\}$

horizontal asymptotes $y = 1$

sign check (+) DNE (−) 0 (+) DNE (−) 0 (+)

$\leftarrow$ − − − | + − − | + − − − | + − − | − − − $\rightarrow$

$-6 \quad -5 \quad 3 \quad 5$

$\lim_{x \to -\infty} f(x) = 1^+$ $\lim_{x \to \infty} f(x) = 1^-$

State the domain and range using **interval notation.**

Domain: $(-\infty, -6) \cup (-6, 3) \cup (3, \infty)$ Range: $(-\infty, \infty)$