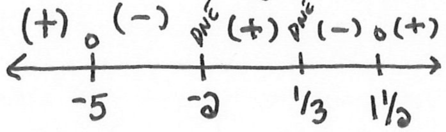
Calculus (in class) Test 1 2013 **Name**________________________

Solve each of the inequalities

#1 Solve the inequality, be sure answer is in **interval notation**.

$$\frac{2x^2+7x-15}{3x^2+5x-2} \le 0 \qquad \frac{(2x-3)(x+5)}{(3x-1)(x+2)} \le 0$$

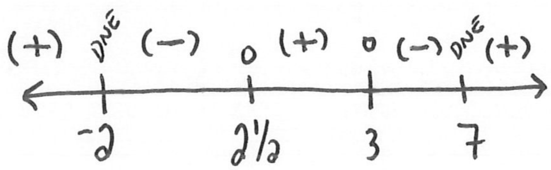
#1 $[-5,-2) \cup (1/3, 1\tfrac{1}{2}]$



#2 Solve the inequality, be sure answer is in **interval notation**.

$$\frac{(g-3)(2g-5)}{(g-7)(g+2)} \le 0$$

#2 $(-2, 2\tfrac{1}{2}] \cup [3,7)$



Functions for problem #3

$f(x) = 2x-7$, $g(x) = x^2 - 4x - 5$, $h(x) = x^2 + 4$

a. find f (9) + g (2) + h (0) #3a 6

$f(9) = 2(9)-7 = 18-7 = 11$

$g(2) = 2^2-4(2)-5 = 4-8-5 = -9$

$h(0) = 0^2+4 = 0+4 = 4$

b. find f (f (3)) #3b -9

$f(3) = 2(3)-7 = 6-7 = -1 \rightarrow f(-1) = 2(-1)-7 = -2-7 = -9$

c. find (g (h (f (2))) #3c 112

$f(2) = 2(2)-7 = 4-7 = -3 \rightarrow h(-3) = (-3)^2+4 = 9+4 = 13 \rightarrow g(13) = 13^2-4(13)-5 = 169-52-5 = 112$

d. Find the **solution set** for f(g(x)) = g(f(x)) #3d $x = \{4.8787, 9.1213$

(**Four** decimal places)

L.H.S. R.H.S.

$$2[x^2-4x-5]-7 = [2x-7]^2 - 4[2x-7]-5$$

$$2x^2-8x-10-7 = 4x^2-28x+49-8x+28-5$$

$$2x^2-8x-17 = 4x^2-36x+72$$

$$0 = 2x^2-28x+89$$

→ USE QUADRATIC FORMULA

$x = \{4.8787, 9.1213\}$

#4 Identify the **center** and the **radius**.

$x^2 + y^2 - 4x + 6y - 3 = 0$

$x^2 - 4x + \underline{4} + y^2 + 6y + \underline{9} = 3 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 16$

#4 CENTER $(2,-3)$ RADIUS: 4

#8 Solve the following systems of equations

ENTER INTO MATRIX EDITOR THEN USE RREF OPTION

a. $x + 3y - z = 13$
$x + 2y + 3z = 4$
$2x - 3y + 4z = 5$

b. $6x + 2y = 34$
$3x - 4y = 7$

c. $3x - y + 2z = 16$
$x + 7y - 4z = -19$
$5x - 2y + 2z = 27$

a $(8, 1, -2)$

b $(5, 2)$

c $(4, -3, \tfrac{1}{2})$

#9 Write the equation of each circle
$(1, -5)$
$(2, -6)$
$(8, 2)$

USE GENERAL FORM

$x^2 + Ax + y^2 + By + C = 0$

$(1,-5)$: $1 + A + 25 - 5B + C = 0$

$A - 5B + C = -26$

$(2,-6)$: $4 + 2A + 36 - 6B + C = 0$

$2A - 6B + C = -40$

$(8,2)$: $64 + 8A + 4 + 2B + C = 0$

$8A + 2B + C = -68$

SYSTEM

$$\begin{bmatrix} 1 & -5 & 1 & -26 \\ 2 & -6 & 1 & -40 \\ 8 & 2 & 1 & -68 \end{bmatrix} \Rightarrow \begin{matrix} A = -10 \\ B = 4 \\ C = 4 \end{matrix}$$

$(x-5)^2 + (y+2)^2 = 25$

$x^2 - 10x + 25 + y^2 + 4y + 4 = -4 + 29$

$(x-5)^2 + (y+2)^2 = 25$

#10 Solve **(Answer as mixed numbers no decimals please)**

a) $10|3x+5|^2 - 11|3x+5| = -3$

#12a $x = \{-1\tfrac{7}{15}, -1\tfrac{1}{2}, -1\tfrac{5}{6}, -1\tfrac{13}{15}\}$

LET $k = |3x+5|$

SO $10k^2 - 11k + 3 = 0$

$(2k-1)(5k-3) = 0$

THEN $k = \frac{1}{2}, \frac{3}{5}$

$|3x+5| = \frac{1}{2}$

$3x+5 = \frac{1}{2}$ $\quad$ $3x+5 = -\frac{1}{2}$

$3x = \frac{1}{2} - \frac{10}{2}$ $\quad$ $3x = \frac{-1}{2} - \frac{10}{2}$

$3x = \frac{-9}{2}$ $\quad$ $3x = \frac{-11}{2}$

$x = \frac{-3}{2}$ $\quad$ $x = \frac{-11}{6}$

$|3x+5| = \frac{3}{5}$

$3x+5 = \frac{3}{5}$ $\quad$ $3x+5 = \frac{-3}{5}$

$3x = \frac{3}{5} - \frac{25}{5}$ $\quad$ $3x = \frac{-3}{5} - \frac{25}{5}$

$3x = \frac{-22}{5}$ $\quad$ $3x = \frac{-28}{5}$

$x = \frac{-22}{15}$ $\quad$ $x = \frac{-28}{15}$

#11 Write each of the following in standard H , K forms identify characteristics of each

a. $25x^2 + 4y^2 - 150x + 8y + 129 = 0$

$$25[x^2 - 6x + 9] + 4[y^2 + 2y + 1] = -129 \quad +225 + 4$$

$$25(x-3)^2 + 4(y+1)^2 = 100$$

#11a. $\dfrac{(x-3)^2}{4} + \dfrac{(y+1)^2}{25} = 1$

Characteristics: VERTICAL ELIPSE

CENTER (3,-1)

MAJOR ENDPOINTS (3,-6) & (3,4)

MINOR END POINTS (1,-1) & (5,-1)

b. $x^2 + 8x - 2y + 2 = 0$

$$x^2 + 8x + 2 = 2y$$

$$(x^2 + 8x + \underline{16}) + 2 - \underline{16} = 2y$$

$$(x+4)^2 - 14 = 2y$$

$$\tfrac{1}{2}(x+4)^2 - 7 = y$$

#11b. $y = \tfrac{1}{2}(x+4)^2 - 7$

Characteristics: PARABOLA

VERTEX (-4,-7)

OPENS UPWARD

OBTUSE OPENING

$\Delta X = 2$

#12. Find the area of of the triangle with vertices at (3 , 1), (–5 , –2), and (11 , –3)

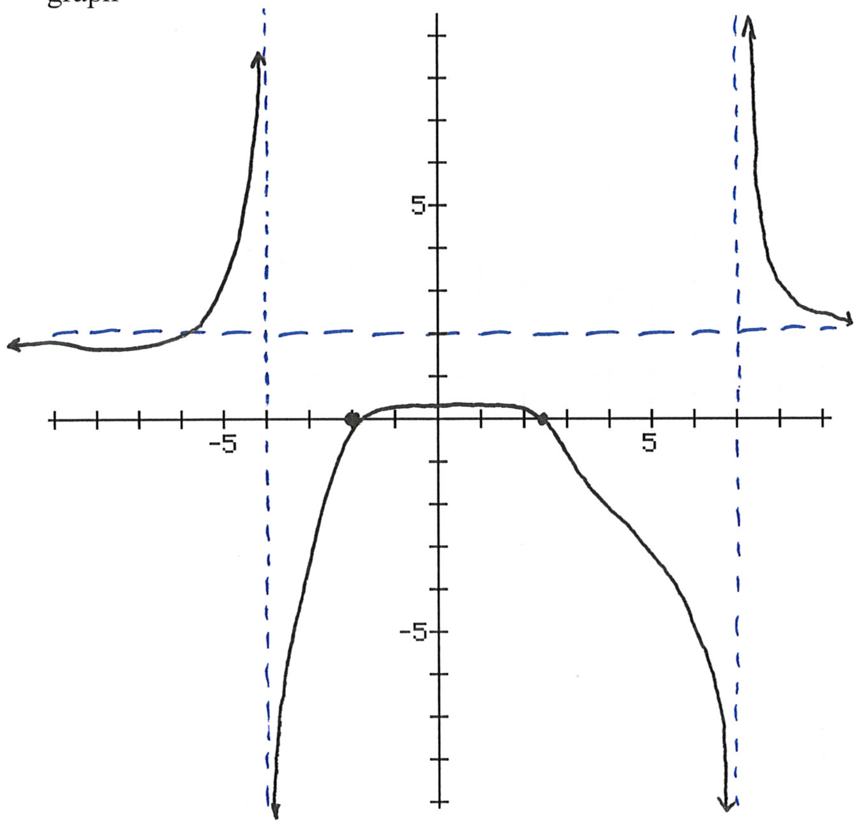
$$\frac{1}{2}\begin{vmatrix} 3, & 1, & 1 \\ -5, & -2, & 1 \\ 11, & -3, & 1 \end{vmatrix} = \frac{1}{2}|56| = 28 \text{ UNITS}$$

#12 28 UNITS

Graph and identify zeroes, all asypmtotes, and perform a sign check.

13a. $y = \dfrac{2x^2 - x - 10}{x^2 - 3x - 28} = \dfrac{(2x-5)(x+2)}{(x-7)(x+4)}$

graph



zeroes $x = \{-2, 2\frac{1}{2}\}$

vertical asymptotes $x \neq \{-4, 7\}$

horizontal asymptotes $y = 2$

sign check

(+)	DNE -4	(−)	0 -2	(+)	0 $2\frac{1}{2}$	(−)	DNE 7

$\lim_{x \to -\infty} f(x) = 2^-$ $\lim_{x \to \infty} f(x) = 2^+$

13b State the domain and range using **interval notation.**

13b Domain: $(-\infty, -4) \cup (-4, 7) \cup (7, \infty)$ 13b Range: $(-\infty, .3572] \cup [1.8742, \infty)$

* USE MAX / MIN OPTION ON GRAPH TO IDENTIFY THESE VALUES

Use three decimal places in calculations, draw a picture, organize your work. I expect a professional presentation of your work for each problem.

"The Great Escape"

1. A hot air balloon carries a cannon straight up at a rate of (5 ft/sec) from a prison yard. Once the cannon has reached exactly 73 ft an automatic firing mechanism is engaged. An ambitious inmate has set herself up to be a human cannon ball. Her hope is that the scheme will allow for her to be shot over the 20 ft wall that is situated 112 ft from where the balloon was launched. She set the cannon to shoot at an upward angle of 35° from the horizontal at a rate of (48 ft/sec). **Will she clear the wall if the prison is located on Saturn?**

"Catapult or Kong"

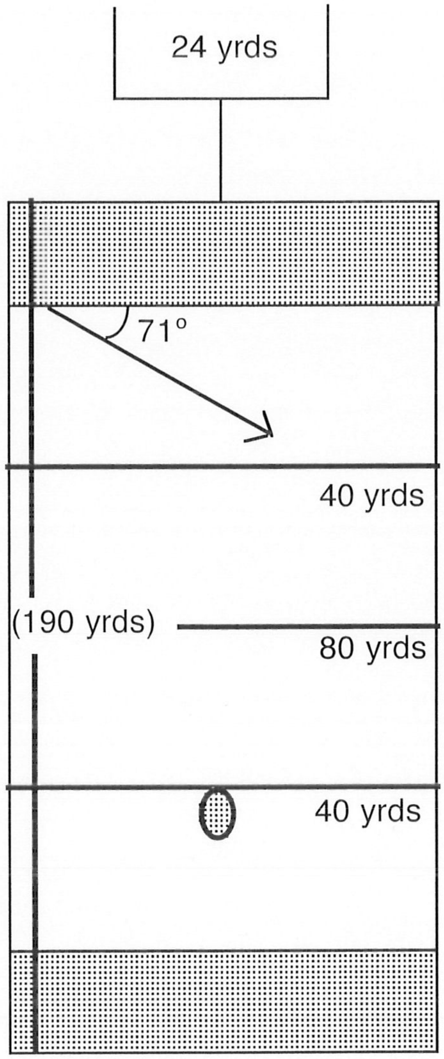
2a. You have joined the rebel forces revolting against the King on Jupiter. You have designed a catapult to heave massive boulders at King Kong's fortress of Doom. The idea is that if the walls can be compromised the rebel forces can overtake the fortress. Any launch that travels at least 135 meters but less that 150 meters should cause damage and hence be considered successful. In other words, for this problem it does not matter if the boulder hits the wall, goes over the wall, or hits any other structure found within the fortress as long as it travels between 135 and 150 meters along the horizontal. The catapult is situated on a small hill that rises up 21 meters from ground level. When the catapult is fired it will launch the projectile from an additional height of 13 meters at an upward angle of 43° from the horizontal with a rate of (50 m/sec). **Will the projectile land within the confines of the fortress?**

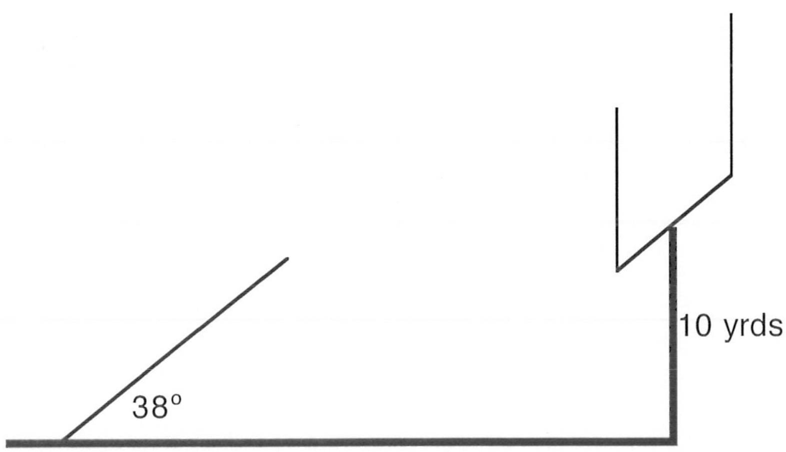
2b. What if the catapult were to be moved off the hill, but 40 meters closer to the fortress will the result change? (Remember that by moving the catapult 40 m, one will lose height and reduce the distance that the projectile needs to travel.)

"An Out-Of-This-World Field Goal"

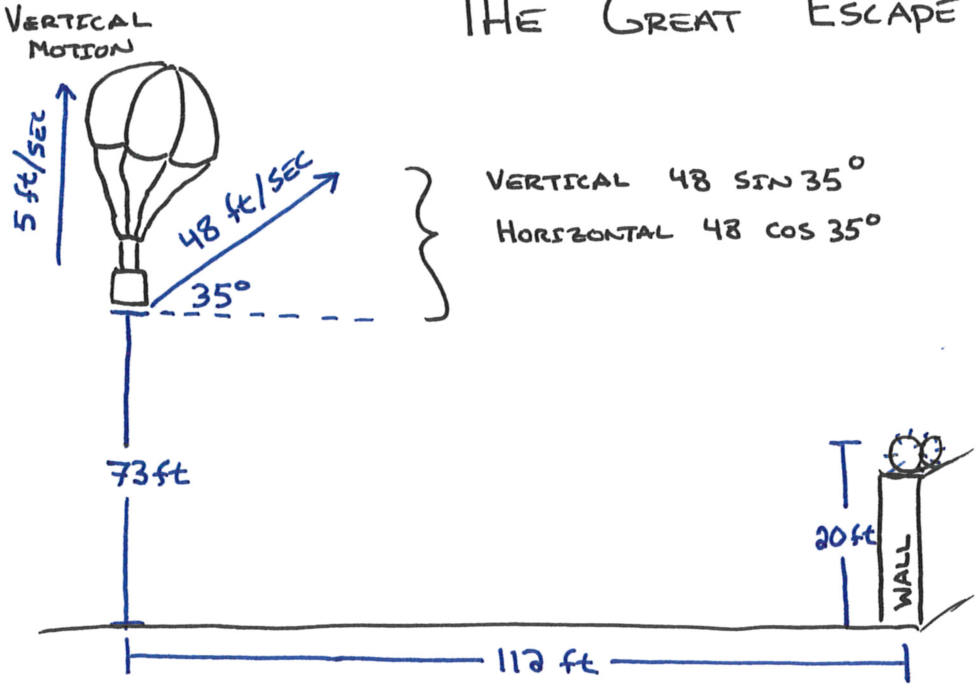
3. The scenario is that a group of oversized aliens are true fans of the NCAA Bowl Games and have decided to hold their championship at the first ever Neptune Bowl. Because of their girth and exceptional strength They have made a few modifications to the standard size football field. They have extended the field to **160 yrds** with endzones each measuring and additional **15 yrds**. The Goal Posts are set **10 yrds** up in the air and open **24 yrds.** Mickey Stonefoot is about to attempt a field goal from his own 40 yard line. There is a modest wind breaking 71° South of West at **17 mph.** If Mickey launches his attempt at an angle of 38° to the ground aiming straight down the pike with a velocity of **154 ft/sec** will he make the field goal? I have provided some diagrams that hopefully help understand the setup.

Unfortunately, I must complicate the issue. For argument sake lets say that the ball travels well beyond the goal post. Then the time that the ball reaches the height of the goal post will have then in effect given extra time to the "shank" calculation. In other words, If the ball travels well beyond the goal post then one will want to figure out the exact time the ball actually passes through the post. That moment of time should be used to calculate how far the ball has drifted to the right. Only then can one determine if the ball passes through the plane of the goal post.


24 yrds
71°
40 yrds
(190 yrds)
80 yrds
40 yrds


10 yrds
38°

"THE GREAT ESCAPE"



VERTICAL 48 SIN 35°

HORIZONTAL 48 COS 35°

GRAVITY FOR SATURN $g = \dfrac{G \cdot M}{R^2} = \dfrac{[6.67 \times 10^{-11}][5.69 \times 10^{26}]}{[6.03 \times 10^{7}]^2}$

$= 10.438 \ m/s^2$ OR $34.236 \ ft/sec^2$

CONSIDER VERTICAL MOTION

$$S(t) = \tfrac{1}{2}at^2 + V_0 t + h_0$$

$$S(t) = \tfrac{1}{2}(-34.236)t^2 + [5 + 48 \sin 35°]t + 73$$

GRAVITY WILL BE NEGATIVE ACCELERATION

VELOCITY
5 ft/sec FROM BALLOON
48 SIN 35° FROM CANNON

HEIGHT
73 FROM BALLOON

SET S(t) = 20 ft (HEIGHT OF WALL)

$$20 = \tfrac{1}{2}(-34.236)t^2 + [5 + 48 \sin 35°]t + 73$$

$$0 = \tfrac{1}{2}(-34.236)t^2 + [5 + 48 \sin 35°]t + 53$$

USE QUADRATIC FORMULA

$$t = \{-1.050, \ 2.950\}$$

SO SHE WILL REACH HEIGHT OF 20 ft FROM GROUND AT $t = 2.950$

WILL SHE HAVE TRAVELED ENOUGH HORIZONTALLY TO CLEAR THE WALL?

$D = r \cdot t$

METHOD 1
COMPARE TIMES

$112 = 48 \cos 35° \cdot t$

$2.848 = t$

∴ PASSES WALL BEFORE REACHING HEIGHT OF 20 ft

METHOD 2
COMPARE DISTANCES

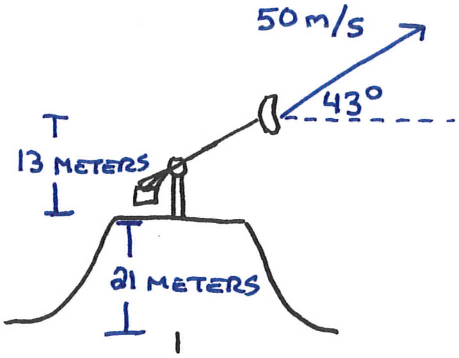
$D = [48 \cos 35°](2.950)$

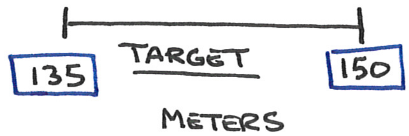
$D = 115.992 \ ft$

∴ REACHES HEIGHT OF WALL AFTER TRAVELING THE MINIMUM DISTANCE OF 112 ft

HENCE, SHE CLEARED THE WALL

"Catapult or Kong" Part I





Gravity for Jupiter

$$g = \frac{G \cdot M}{R^2} = \frac{[6.67 \times 10^{-11}][1.90 \times 10^{27}]}{[7.15 \times 10^{7}]^2} = 24.789\ m/s^2$$

Consider Vertical Motion

$$S(t) = \tfrac{1}{2}(-24.789)t^2 + [50 \sin 43^\circ]t + 34$$: Everything in meters

$$0 = \tfrac{1}{2}(-24.789)t^2 + [50 \sin 43^\circ]t + 34$$ ∵ Concern for impact at ground level.

Use Quadratic Formula

$$t = \{-.777,\ 3.529\}$$

So "Air time" is 3.529 seconds

Now Consider Horizontal Motion

$$D = r \cdot t$$

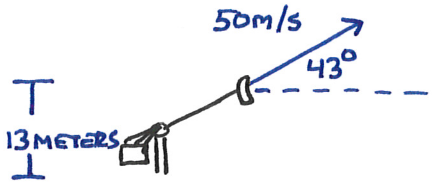
$$D = [50 \cdot \cos 43^\circ][3.529]$$ ∵ Horizontal velocity with 3.529 seconds of air time.

$$D = 129.047 \text{ meters}$$

∴ Missed the Target

Projectile does not travel far enough.

"CATAPULT OR KONG" PART II



ADJUST DISTANCE BECAUSE CATAPULT MOVED CLOSER.
∴ NO LONGER ON HILL
∴ CONCERN FOR IMPACT AT GROUND LEVEL $S(t)=0$

CONSIDER VERTICAL MOTION

$$S(t) = \frac{1}{2}(-24.789)t^2 + 50\sin 43^\circ + 13$$

$$0 = \frac{1}{2}(-24.789)t^2 + 50\sin 43^\circ + 13$$

USE QUADRATIC FORMULA

$$t = \{-.339,\ 3.091\}$$

SO "AIR TIME" IS 3.091 SECONDS

NOW CONSIDER HORIZONTAL MOTION

$$D = r \cdot t$$

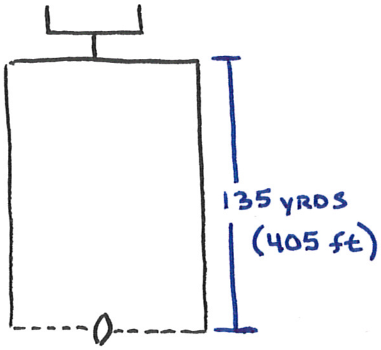
$$D = [50\cos 43^\circ][3.091]$$

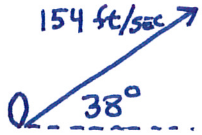
$$D = 113.031 \text{ METERS}$$

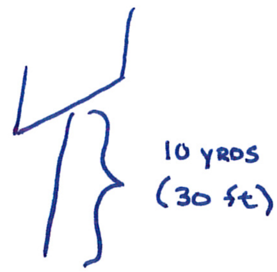
∴ MISSED THE TARGET

PROJECTILE TRAVELS TOO FAR.

"An Out-Of-This-World Field Goal"







Gravity For Neptune

$$g = \frac{G \cdot M}{R^2} = \frac{[6.67 \times 10^{-11}][1.03 \times 10^{26}]}{[2.48 \times 10^{7}]^2} = 11.170 \text{ m/s}^2$$

$$= 36.638 \text{ ft/s}^2$$

Consider Vertical Motion

$$S(t) = \tfrac{1}{2} a t^2 + V_0 t + h_0$$

$$S(t) = \tfrac{1}{2}(-36.638)t^2 + [154 \sin 38°]t$$

} : Gravity Negative
: Everything in feet
: Vertical velocity for the kick (from ground)

Set $S(t) = 30$ ft Expect to see two times where ball reaches this height.

$$0 = \tfrac{1}{2}(-36.638)t^2 + [154 \sin 38°]t - 30$$

Use quadratic formula

$$t = \{.339,\ 4.837\}$$

I. Does the ball at least break the plane of the field goal posts?

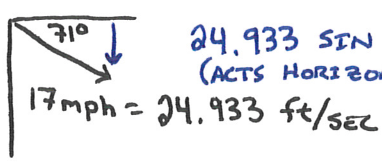
Consider Horizontal Motion

$D = r \cdot t$, But horizontal velocity is broken down into two components

A. Kicking Vector

$154 \cos 38°$

B. Wind Vector



$24.933 \sin 71°$
(acts horizontally on field)

$$D = [154 \cos 38° - 24.933 \sin 71°]\, 4.837$$

$$D = 472.957 \text{ ft}$$

∴ The ball has enough distance to break the plane of the goal posts, But is it accurate enough?

II. DETERMINE WHERE THE BALL IS WHEN IT BREAKS THE PLANE OF THE GOAL POSTS.

FIRST, CONSIDER WHEN IT BREAKS THE PLANE.

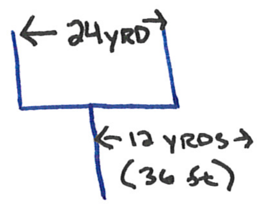
$D = r \cdot t$: HORIZONTAL MOTION

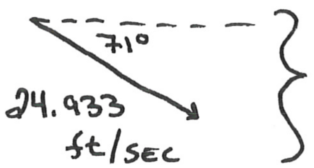
$405 = [154 \cos 38^\circ - 24.933 \sin 71^\circ]\, t$

$4.142 \text{ seconds} = t$: TIME FOR BALL TO BE ON THE FIELD GOAL POST PLANE

NEXT, CONSIDER HOW MUCH THE BALL HAS DRIFTED DUE TO THE WIND.

THE BALL STARTED ON CENTER WITH THE FIELD GOAL SO AT MOST IT CAN DRIFT IS 36 ft IF THE FIELD GOAL IS MADE.





DRIFTING DISTANCE = RATE · TIME

$D = [24.933 \cos 71^\circ]\ 4.142$

$D = 33.622 \text{ ft}$

$\therefore$ THE BALL IS INSIDE THE POSTS AND HAS ENOUGH HEIGHT AT THE TIME THAT THE BALL BREAKS THE PLANE, HENCE THE FIELD GOAL IS GOOD!