Calculus (Review) **Name** KEY

Solve each of the inequalities

#1 Solve the inequality, be sure answer is in <u>interval notation</u>.

$$\frac{(3x+10)}{(x-3)(x+7)} \geq 0$$

(−) DNE (+) 0 (−) DNE (+)

−7 $-3\frac{1}{3}$ 3

: PERFORM SIGN TEST ON ZEROS AND RESTRICTIONS

SOLUTION: $(-7, -3\tfrac{1}{3}] \cup (3, \infty)$

#2 Solve the inequality, be sure answer is in interval notation.

$$\frac{k^2+2k-8}{k^2-10k+21} \leq 0 \qquad \frac{(k+4)(k-2)}{(k-3)(k-7)} \leq 0$$

(+) 0 (−) 0 (+) DNE (−) DNE (+)

−4 2 3 7

: PERFORM SIGN TEST ON ZEROS AND RESTRICTIONS

SOLUTION $[-4, 2] \cup (3, 7)$

Functions for problems 3 and 4 $f(x) = 3x^2 + x - 2$ and $g(x) = -2x^2 + 3x - 1$ and $h(x) = 3x + 1$

#3 Find and simplify $g(f(h(0)))$

$g(f(1))$

$\boxed{g(2) = -3}$

$h(0) = 3(0) + 1$
$= 1$

$f(1) = 3(1^2) + 1 - 2$
$= 2$

$g(2) = -2(2^2) + 3(2) - 1$
$= -8 + 6 - 1$
$= -3$

#4 Find and simplify $f(g(-3))$

$\boxed{f(-28) = 2322}$

$g(-3) = -2(-3^2) + 3(-3) - 1$
$= -18 - 9 - 1$
$= -28$

$f(-28) = 3(-28^2) - 28 - 2$

#5 Find and simplify $h(g(h(-3)))$

$h(g(-8))$

$\boxed{h(-153) = -458}$

$h(-3) = 3(-3) + 1$
$= -9 + 3$
$= -8$

$g(-8) = -2(-8^2) + 3(-8) - 1$
$= -128 - 24 - 1$
$= -153$

$h(-153) = 3(-153) + 1$
$= -458$

#6 Identify the center and the radius.

a) $x^2 + y^2 - 4x + 2y - 31 = 0$

b) $x^2 + y^2 + 6x - 10y - 6 = 0$

a) $x^2 - 4x \quad + y^2 + 2y \quad = 31$

$x^2 - 4x + 4 \quad + y^2 + 2y + 1 = 31 + 4 + 1$

$(x-2)^2 + (y+1)^2 = 36$

CENTER: $(2,-1)$ RADIUS: 6

b) $x^2 + 6x \quad + y^2 - 10y \quad = 6$

$x^2 + 6x + 9 \quad + y^2 - 10y + 25 = 6 + 9 + 25$

$(x+3)^2 + (y-5)^2 = 40$

CENTER $(-3,5)$ RADIUS: $2\sqrt{10}$

7 Solve the systems.

a)
$$\begin{aligned} 3x + 6y - 6z &= 9 \\ 2x - 5y + 4z &= 6 \\ -x + 16y + 14z &= -3 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$(3,0,0)$

b)
$$\begin{aligned} 2.5x + y - z &= -6 \\ -3.5y + 2.5z &= 2.5 \\ 5x + 4y - 2z &= -12 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$(-2,0,1)$

c)
$$\begin{aligned} x + y - 2z &= -2 \\ 2x - 3y + z &= 1 \\ 2x + y - 3z &= -2 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

NO SOLUTION

#8 Find the equation of the circle that contains these points.

$(-8,-4)$
$(-7,-5)$
$(-3,\ 1)$

$x^2 + y^2 + Ax + By + C = 0$

$64 + 16 - 8x - 4y + C = 0$

$49 + 25 - 7x - 5y + C = 0$

$9 + 1 - 3x + y + C = 0$

$-8x - 4y + C = -80$

$-7x - 5y + C = -74$

$-3x + y + C = -10$

$A = 10$
$B = 4$
$C = 16$

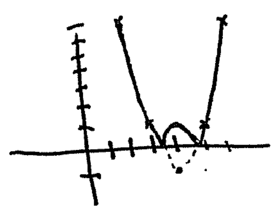
$x^2 + 10x \quad + y^2 + 4y \quad = -16$

$x^2 + 10x + 25 \quad + y^2 + 4y + 4 = -16 + 2$

$(x+5)^2 + (y+2)^2 = 13$

9 Graph and state the domain and range using **interval notation.**

a) $y = |2(x-4)^2 - 1|$



D: $(-\infty, \infty)$
R: $[0, \infty)$

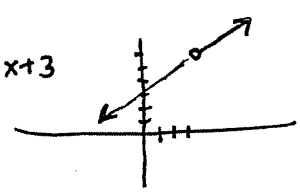
b) $y = \dfrac{x^2 - 9}{x - 3}$

$\dfrac{(x-3)(x+3)}{x-3}$

ZERO $x = -3, 3$
RESTRICTION $x = 3$ } HOLE @ $x=3$

D: $(-\infty, 3) \cup (3, \infty)$
R: $(-\infty, 6) \cup (6, \infty)$

$y = x + 3$



#11 Write each of the following in standard H , K forms identify characteristics of each

a. $4x^2+4y^2-4x+8y=11$

$x^2-x \quad +y^2+2y \quad = \frac{11}{4}$: DIVIDE BY 4

$x^2-x+\frac{1}{4} \quad +y^2+2y+1 = \frac{11}{4}+\frac{1}{4}+1$: COMPLETE SQUARES & BALANCE EQUATION

$(x-\frac{1}{2})^2+(y+1)^2=4$: FACTOR

$\boxed{\text{CIRCLE: CENTER } (1/2,-1) \quad \text{RADIUS} = 2}$

b. $9x^2+16y^2-18x-64y-71=0$

$9x^2-18x \quad +16y^2-64y \quad = 71$: SEPARATE LIKE TERMS

$9[x^2-2x+1]+16[y^2-4y+4] = 71+9+64$: COMPLETE SQUARES & BALANCE EQ.

$9(x-1)^2+16(y-2)^2=144$: FACTOR THEN DIVIDE BY COEFFICIENTS

$\frac{(x-1)^2}{16}+\frac{(y-2)^2}{9}=1$

ELLIPSE: CENTER (1, 2)
HORIZONTAL MAJOR AXIS LENGTH 8, END POINTS $(1\pm4, 2)$
VERTICAL MINOR AXIS LENGTH 6, ENDPOINTS $(1, 2\pm3)$
FOCI $(1\pm\sqrt{7}, 2)$

c. $3x^2-6x-6y+10=0$

$3x^2-6x \quad +10 = 6y$

$3[x^2-2x+1]-3+10=6y$: COMPLETE SQUARE & BALANCE EQ.

$3(x-1)^2+7=6y$: FACTOR, NEXT DIVIDE BY 6

$\frac{1}{2}(x-1)^2+\frac{7}{6}=y$

PARABOLA: VERTEX $(1, 1\frac{1}{6})$
OBTUSE OPENING
OPENS UPWARD
$\Delta x = 2$

#12 Solve

a. $3|4x+5|^2-10|4x+5|=-8$

LET $|4x+5| = m$

$3m^2-10m+8=0$

$(3m-4)(m-2)=0$

$m=\frac{4}{3} \quad m=2$

$|4x+5|=\frac{4}{3} \quad |4x+5|=2$

$4x+5=\frac{4}{3} \quad 4x+5=-\frac{4}{3} \quad 4x+5=2 \quad 4x+5=-2$

$4x=-3 \quad 4x=-7$

$x=-\frac{11}{12} \quad x=-1\frac{7}{12} \quad x=-3/4 \quad x=-1\frac{3}{4}$

a. $\boxed{x=\{-1\tfrac{3}{4}, -1\tfrac{7}{12}, -\tfrac{11}{12}, -\tfrac{3}{4}\}}$

b. $3|3x-1|^2-5|3x-1|=2$

LET $|3x-1| = k$

$3k^2-5k-2=0$

$(3k+1)(k-2)=0$

$k=-\frac{1}{3} \quad k=2$

$|3x-1|=-\frac{1}{3}$ NONE $\quad |3x-1|=2$

$3x-1=2 \quad 3x-1=-2$

b. $\boxed{x=\{-1/3, 1\}}$

#13. Find the area of of the triangle with vertices at (-6 , 5), (-2 , -3), and (4 , 7)

$$\frac{1}{2}\begin{vmatrix} -6 & 5 & 1 \\ -2 & -3 & 1 \\ 4 & 7 & 1 \end{vmatrix} = \frac{1}{2}(88) = 44$$

#14. 7) If $f(x) = 2x + 4$, $g(x) = \sqrt{x^2 - 3} + 1$, $h(x) = x^2 + 9$

a. find $f(3) + g(\sqrt{3}) + h(0)$

$10 + 1 + 9$

$\boxed{20}$

$f(3) = 2(3)+4$
$= 10$

$g(\sqrt{3}) = \sqrt{(\sqrt{3})^2 - 3} + 1$
$= 1$

$h(0) = 9$

b. find $f(f(4))$

$f(12)$

$\boxed{28}$

$f(4) = 2(4)+4$
$= 12$

$f(12) = 2(12)+4$
$= 28$

c. find $(g(f(h(1)))$

$g(f(10))$

$g(24) = \boxed{\sqrt{573} + 1}$

$h(1) = 1^2 + 9$
$= 10$

$f(10) = 2(10)+4$
$= 24$

#15. Given the function $g(x) = \dfrac{3x}{x-6} + 9$, show what g (g (x)) would equal in simplist form.

$$g(g(x)) = \frac{3\left[\frac{3x}{x-6} + 9\right]}{\left[\frac{3x}{x-6} + 9\right] - 6} + 9$$

$$= \frac{3\left[\frac{3x}{x-6} + \frac{9(x-6)}{x-6}\right]}{\frac{3x}{x-6} + \frac{3(x-6)}{x-6}} + 9$$

$$= \frac{3\left[\frac{12x - 54}{x-6}\right]}{\frac{6x-18}{x-6}} + 9$$

$$= 3\left[\frac{6(2x-9)}{x-6} \cdot \frac{x-6}{6(x-3)}\right] + 9$$

$$= \frac{3(2x-9)}{x-3} + 9$$

$$= \frac{3(2x-9)}{x-3} + \frac{9(x-3)}{x-3}$$

$$= \frac{6x - 27 + 9x - 27}{x-3}$$

$$= \frac{15x - 54}{x-3}$$

$$= \frac{15x - 45 - 9}{x-3}$$

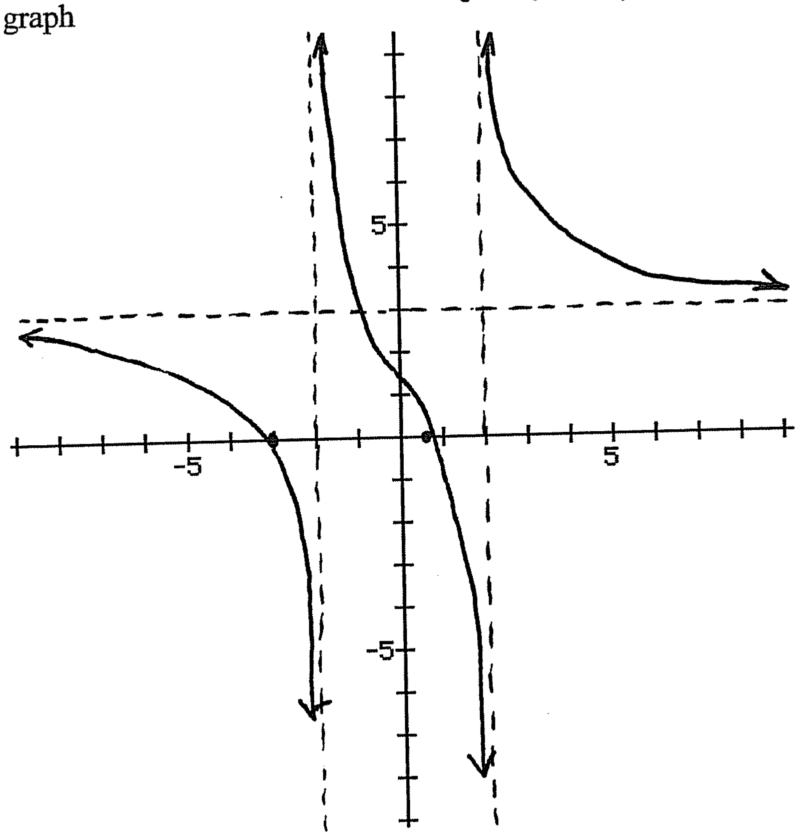
$$= \frac{15(x-3) - 9}{(x-3)}$$

$$= 15 - \frac{9}{x-3}$$

Graph and identify zeroes, all asypmtotes, and perform a sign check, remember to check extreme values.

16. $y = \dfrac{3x^2 + 7x - 6}{x^2 - 4}$ $\qquad \dfrac{(3x-2)(x+3)}{(x-2)(x+2)}$

graph



zeroes $x = 2/3 \quad x = -3$

vertical asymptotes $x = -2 \quad x = 2$

horizontal asymptotes $y = 3$

sign check

(+) 0 (−) DNE (+) 0 (−) DNE +

$-3 \quad -2 \quad \frac{2}{3} \quad 2$

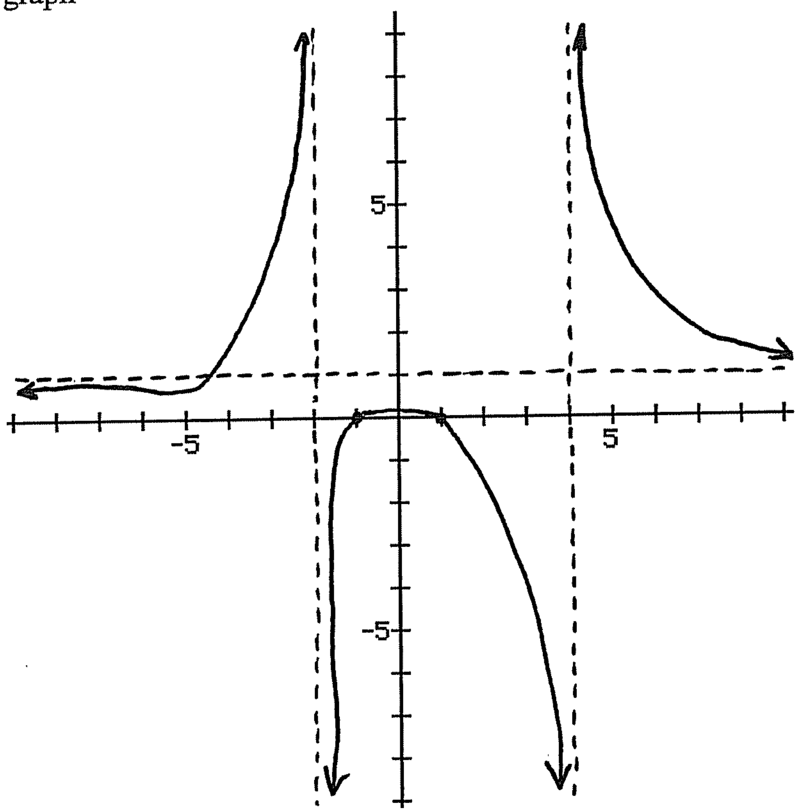
$\lim_{x \to -\infty} f(x) = 3^-$

$\lim_{x \to \infty} f(x) = 3^+$

Graph and identify zeroes, all asypmtotes, and perform a sign check.

17. $\dfrac{x^2-1}{x^2-2x-8} = \dfrac{(x-1)(x+1)}{(x-4)(x+2)}$

graph



zeroes $x=-1,\ x=1$

vertical asymptotes $x=-2,\ x=4$

horizontal asymptotes $y=1$

sign check

(+)	DNE	(−)	0	(+)	0	(−)	DNE	(+)
	−2		−1		1		4	

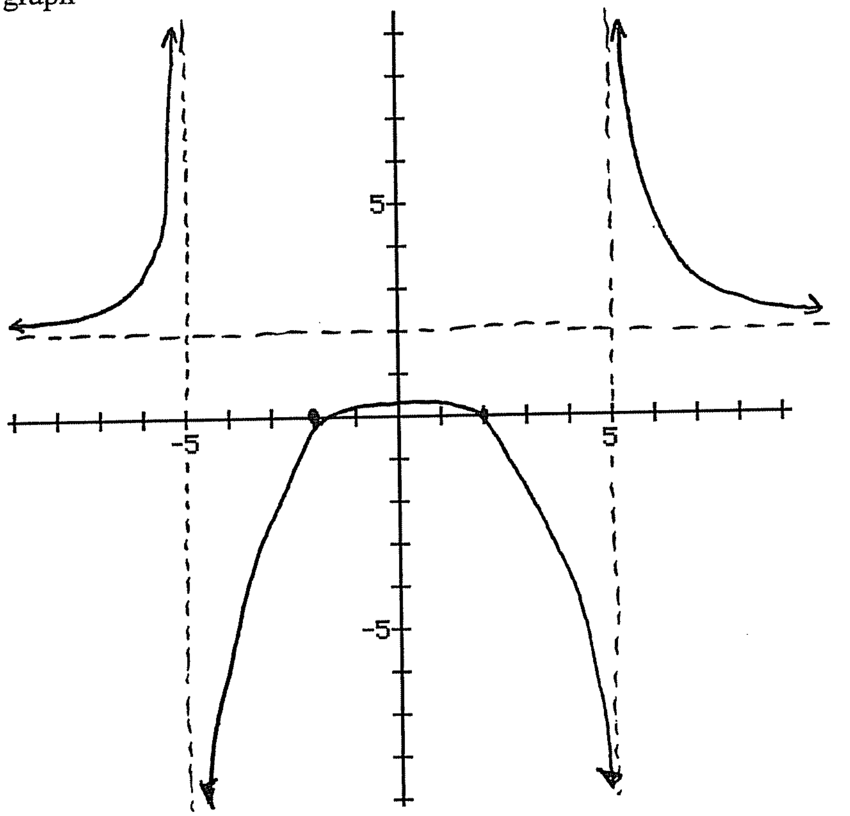
$\lim_{x\to-\infty} f(x) = 1^-$

$\lim_{x\to\infty} f(x) = 1^+$

Graph and identify zeroes, all asypmtotes, and perform a sign check.

18. $\dfrac{2x^2-8}{x^2-25} = \dfrac{2(x-2)(x+2)}{(x-5)(x+5)}$

graph



zeroes $x=-2$, $x=2$

vertical asymptotes $x=-5$, $x=5$

horizontal asymptotes $y=2$

sign check

(+)	DNE	(−)	0	(+)	0	(−)	DNE	(+)
	-5		-2		2		5	

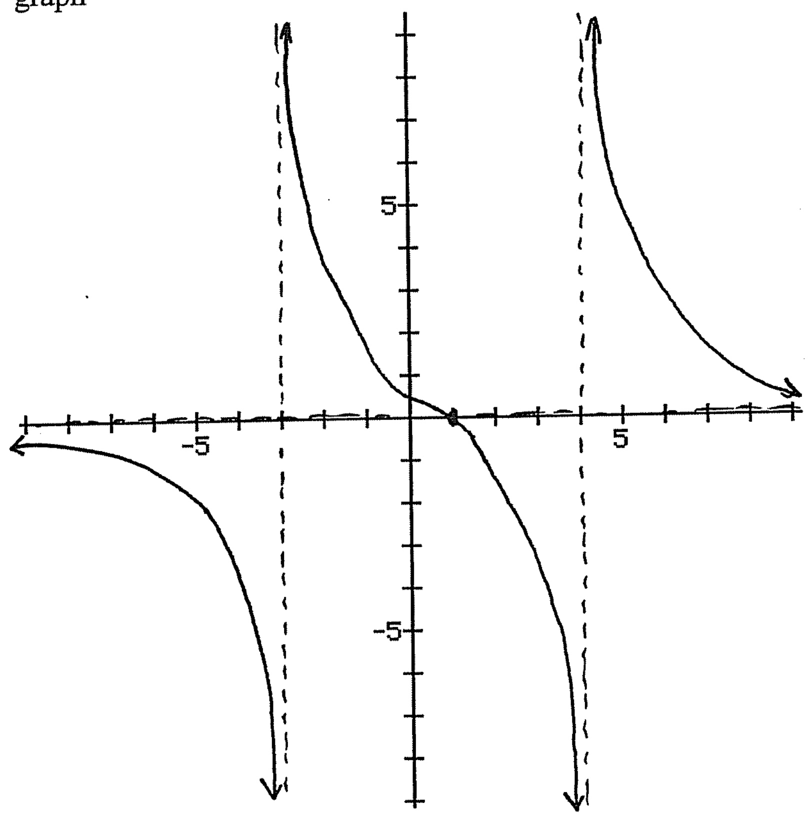
$\lim_{x\to-\infty} f(x)=2^+$

$\lim_{x\to\infty} f(x)=2^+$

Graph and identify zeroes, all asypmtotes, and perform a sign check.

19. $\dfrac{7x-7}{x^2-x-12} = \dfrac{7(x-1)}{(x-4)(x+3)}$

graph



zeroes $x=1$

vertical asymptotes $x=-3$ AND $x=4$

horizontal asymptotes $y = \frac{7}{x} = 0$ (APPROACHES X-AXIS)

sign check

(−)	DNE	(+)	0	(−)	DNE	(+)
	−3		1		4	

$\lim\limits_{x\to-\infty} f(x) = 0^-$

$\lim\limits_{x\to\infty} f(x) = 0^+$