REVIEW

#1 $y = (3x^2+1)^4$

$\frac{dy}{dx} = 4(3x^2+1)^3 \cdot 6x$: WITH CHAIN

$= 24x\,(3x^2+1)^3$

#2

$f(x) = x^2(x-2)^4$

LET $u = x^2$ $v = (x-2)^4$

$du = 2x$ $dv = 4(x-2)^3 \cdot 1$

PRODUCT RULE

$f'(x) = x^2 \cdot 4(x-2)^3 \cdot 1 + (x-2)^4 \cdot 2x$

$f'(x) = 2x\,(x-2)^3\,[2x + x-2]$

$= 2x\,(x-2)^3(3x-2)$

#3 $f(x) = \frac{5}{(x^2-3x)^3}$

$f(x) = 5(x^2-3x)^{-3}$: WRITE AS A POWER RULE

$f'(x) = -15(x^2-3x)^{-4} \cdot [2x-3]$: WITH CHAIN

$= \frac{-15\,(2x-3)}{(x^2-3x)^{-4}}$

#4 $h(r) = \frac{7}{\sqrt[4]{3r^3-4r^2+9}}$

$h(r) = 7\,(3R^3-4R^2+9)^{-1/4}$: WRITE AS POWER RULE

$h'(r) = \frac{-7}{4}(3R^3-4R^2+9)^{-5/4} \cdot (9R^2-8R)$: WITH CHAIN

$= \frac{-7\,(9R^2-8R)}{4\,(3R^3-4R+9)^{5/4}}$

#5 $f(t) = 3t^{2/3} + 5t^{1/2} - 4t + 13$

$f'(t) = 2t^{-1/3} + \frac{5}{2}t^{-1/2} - 4$: POWER RULE

$f'(t) = \frac{2}{t^{1/3}} + \frac{5}{2t^{1/2}} - 4$

#6 $y = \frac{1}{\sqrt{x^3+2}}$

$y = (x^3+2)^{-1/2}$: WRITE AS POWER

$\frac{dy}{dx} = -\frac{1}{2}(x^3+2)^{-3/2} \cdot 3x^2$: CHAIN RULE

$\frac{dy}{dx} = \frac{-3x^2}{2(x^3+2)^{3/2}}$

#7 $y = \frac{3x^2-5}{2x+1}$

$\frac{dy}{dx} = \frac{(2x+1)\cdot 6x - (3x^2-5)\cdot 2}{(2x+1)^2}$: QUOTIENT RULE

$= \frac{12x^2+6x - 6x^2+10}{(2x+1)^2}$

$= \frac{6x^2+6x+10}{(2x+1)^2}$

#8 $y = 3\tan(4x)$

$y' = 3\sec^2(4x)\cdot 4$ CHAIN RULE (ANGLE)

$= 12\sec^2(4x)$

#9 $y = \sec^3(2x)$

$y = [\sec(2x)]^3$: WRITE AS POWER

$\frac{dy}{dx} = 3[\sec(2x)]^2 \cdot \underline{\sec(2x)\tan(2x)} \cdot 2$: CHAIN TO BRACKET THEN CHAIN TO ANGLE

$= 6\sec^3(2x) \cdot \tan(2x)$

#10 $y = \csc\left(\frac{x}{2} + \frac{\pi}{4}\right)$

$y = \frac{1}{\sin\left(\frac{x}{2} + \frac{\pi}{4}\right)}$: SET UP FOR QUOTIENT RULE

$y = \frac{\sin\left(\frac{x}{2} + \frac{\pi}{4}\right) \cdot 0 - 1 \cdot \cos\left(\frac{x}{2} + \frac{\pi}{4}\right) \cdot \frac{1}{2}}{\sin^2\left(\frac{x}{2} + \frac{\pi}{4}\right)}$: CHAIN RULE FOR ANGLE

$= \frac{-\frac{1}{2}\cos\left(\frac{x}{2} + \frac{\pi}{4}\right)}{\sin^2\left(\frac{x}{2} + \frac{\pi}{4}\right)}$

$= -\frac{1}{2}\csc\left(\frac{x}{2} + \frac{\pi}{4}\right)\cot\left(\frac{x}{2} + \frac{\pi}{4}\right)$

#11 $y = 1 - \cos 2x + 2\cos^2 x$: DOUBLE ANGLE FORMULA $\cos 2u = \cos^2 u - \sin^2 u$

$y = 1 - [\cos^2 x - \sin^2 x] + 2\cos^2 x$

$y = 1 - \cos^2 x + \sin^2 x + 2\cos^2 x$

$y = 1 + \sin^2 x + \cos^2 x$: PYTHAGOREAN IDENTITY $\sin^2 x + \cos^2 x = 1$

$y = 1 + 1$

$y = 2$

$\frac{dy}{dx} = 0$

#12 $y = \underline{\sin^2 x} - \sin 2x \underline{+ \cos^2 x}$: CONSIDER PYTHAGOREAN IDENTITY

$\sin^2 x + \cos^2 x = 1$

$y = 1 - \sin 2x$

$\frac{dy}{dx} = 0 - \cos(2x) \cdot 2$: CHAIN RULE FOR ANGLE

$= -2 \cos 2x$

DERIVATIVES FOR TRANSCENDENTAL FUNCTIONS

#1 $y = \ln(\cos 5x)$

$y' = \frac{1}{\cos(5x)} \cdot [-\sin(5x)] \cdot 5$: DERIVATIVE of ARGUMENT THEN CHAIN FOR ANGLE

$y' = -5 \tan(5x)$

$y'' = -5 \sec^2(5x) \cdot 5$: AGAIN CHAIN RULE FOR ANGLE

$y'' = -25 \sec^2(5x)$

#2 $y = x \cdot \ln(x) - x$

LET $u = x$ $v = \ln x$

$du = 1$ $dv = \frac{1}{x} \cdot 1$

$\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1 - 1$: $u\,dv + v\,du$ THEN DERIVATIVE of $-x$

$= 1 + \ln x - 1$

$\frac{dy}{dx} = \ln x$

$\frac{d^2y}{dx^2} = \frac{1}{x}$

#3 $y = e^{5x^2} \cdot \ln(30x^2)$

LET $u = e^{5x^2}$ $\quad v = \ln(30x^2)$

$du = e^{5x^2} \cdot 10x$ $\quad dv = \frac{1}{30x^2} \cdot 60x$

$= 10e^{5x^2}$ $\quad = \frac{2}{x}$

$$\frac{dy}{dx} = e^{5x^2} \cdot \frac{2}{x} + \ln(30x^2) \cdot 10e^{5x^2}$$: PRODUCT RULE

$$\frac{dy}{dx} = 2e^{5x^2}\left[\frac{1}{x} + 5\ln(30x^2)\right]$$: FACTORING

MORE PRACTICE WITH DERIVATIVES

#1 $y = (3x^4+7)^3(5x^3-4)^5$; PRODUCT RULE W/ CHAIN

$$\frac{dy}{dx} = (3x^4+7)^3 \cdot 5(5x^3-4)^4 \cdot \underline{15x^2} + (5x^3-4)^5 \cdot 3(3x^4+7)^2 \cdot \underline{12x^3}$$

$$= 3x^2(3x^4+7)^2(5x^3-4)^4\left[25(3x^4+7) + 12x(5x^3-4)\right]$$: FACTOR GCF

$$= 3x^2(3x^4+7)^2(5x^3-4)^4\left[75x^4 + 175 + 60x^4 - 48x\right]$$

$$= 3x^2(3x^4+7)^2(5x^3-4)^4\left[135x^4 - 48x + 175\right]$$

#2 $y = \frac{(2x+1)^3}{(3x-4)^5}$

$$\frac{dy}{dx} = \frac{(3x-4)^5 \cdot 3(2x+1)^2 \cdot 2 - (2x+1)^3 \cdot 5(3x-4)^4 \cdot 5}{\left[(3x-4)^5\right]^2}$$

$$= \frac{(3x-4)^4(2x+1)^2\left[6(3x-4) - 25(2x+1)\right]}{(3x-4)^{10}}$$

$$= \frac{(2x+1)^2(18x-24-50x-25)}{(3x-4)^6}$$

$$= \frac{(2x+1)^2(-32x-49)}{(3x-4)^6}$$

#3

$$y = \left[\sin\sqrt{2x^3+11}\right]^3$$

$$y' = 3\left[\sin\sqrt{2x^3+11}\right]^2 \cdot \cos\sqrt{2x^3+11} \cdot \frac{1}{2}(2x^3+11)^{-1/2} \cdot 6x^2$$

$$y' = 3\sin^2\sqrt{2x^3+11}\ \cos\sqrt{2x^3+11} \cdot 3x^2 \cdot \frac{1}{\sqrt{2x^3+11}}$$

$$y' = \frac{9x^2\sin^2\sqrt{2x^3+11}\ \cos\sqrt{2x^3+11}}{\sqrt{2x^3+11}}$$

#4

$$y = 5x^2\tan(3x)$$

LET $u = 5x^2$ $\quad v = \tan(3x)$

$du = 10x$ $\quad dv = 3\sec^2(3x)$

$$\frac{dy}{dx} = 5x^2 \cdot 3\sec^2(3x) + \tan(3x) \cdot 10x$$

$$\frac{dy}{dx} = 15x^2\sec^2(3x) + 10x\tan(3x)$$

EXPONENTIALS

#4

$$y = e^{3x^2-5}$$

$$\frac{dy}{dx} = e^{3x^2-5} \cdot 6x$$

$$\frac{dy}{dx} = 6x\,e^{3x^2-5}$$

#5

$$y = (12x^2-5)^4 \cdot e^{2x^3}$$

LET $u = (12x^2-5)^4$ $\quad v = e^{2x^3}$

$du = 4(12x^2-5)^3 \cdot 24x = 96x(12x^2-5)^3$ $\quad dv = e^{2x^3} \cdot 6x^2 = 6x^2 e^{2x^3}$

$$y' = (12x^2-5)^4 \cdot 6x^2 \cdot e^{2x^3} + e^{2x^3} \cdot 96x(12x^2-5)^3$$

$$y' = 6x \cdot e^{2x^3} \cdot (12x^2-5)^3\left[x(12x^2-5) + 16\right]$$

$$y' = 6x \cdot e^{2x^3}(12x^2-5)\left[12x^3 - 5x + 16\right]$$

#6 $y = \sin\left(4e^{\sin 2x}\right)$

$$\frac{dy}{dx} = \cos\left(4e^{\sin 2x}\right) \cdot 4e^{\sin 2x} \cdot \cos 2x \cdot 2$$

$$\frac{dy}{dx} = 8e^{\sin 2x} \cdot \cos 2x \cdot \cos\left(4e^{\sin 2x}\right)$$

NATURAL Logs

#7 $y = \ln (x^2+3)^4$

$$y' = \frac{1}{(x^2+3)^4} \cdot 4(x^2+3)^3 \cdot 2x$$

$$y' = \frac{8x}{x^2+3}$$

ALTERNATE LOOK

$y = 4\ln(x^2+3)$: RULES of EXPONENTS

$$\frac{dy}{dx} = 4 \cdot \frac{1}{x^2+3} \cdot 2x$$

$$\frac{dy}{dx} = \frac{8x}{x^2+3}$$

#8 $y = \tan(5x^2) \ln[\sin(5x^2)]$

LET $u = \tan(5x^2)$ $\quad v = \ln[\sin(5x^2)]$

$du = \sec^2(5x^2) \cdot 10x$ $\quad dv = \frac{10x}{\sin(5x^2)} \cdot \cos(5x^2)$

$= 10x\sec^2(5x^2)$ $\quad = 10x\cot(5x^2)$

$$y' = \underline{\tan(5x^2) \cdot 10x\cot(5x^2)} + \ln[\sin(5x^2)] \cdot 10x\sec^2(5x^2)$$

$$y' = 10x\left[1 + \sec^2(5x^2) \cdot \ln[\sin(5x^2)]\right]$$

#9 $y = 5^{(x^2+3)^4}$

$\ln y = (x^2+3)^4 \ln 5$: RULES of EXPONENTS WITH ln

$$\frac{1}{y} \cdot y' = \ln 5 \cdot 4(x^2+3)^3 \cdot 2x$$

$$y' = y \cdot \ln 5 \cdot 8x(x^2+3)^3$$

$$y' = 5^{(x^2+3)^4} \cdot \ln 5 \cdot 8x(x^2+3)^3$$

#10 $y = 7x^{7x}$

$\ln y = 7x \ln 7x$: REMEMBER THE RULES FOR EXPONENTS

$\frac{1}{y} \cdot y' = 7x \cdot \frac{1}{7x} \cdot 7 + \ln 7x \cdot 7$: $u\,dv + v\,du$

$\frac{1}{y} \cdot y' = 7 + 7 \ln 7x$

$y' = y \cdot 7(1 + \ln 7x)$

$= 7x^{7x} \cdot 7(1 + \ln 7x)$

$= 49x^{7x}(1 + \ln 7x)$

#11 $y = (2x^3-5)^{11} \cdot \sqrt{4x^2+5}$

LET $u = (2x^3-5)^{11}$ $\quad v = (4x^2+5)^{1/2}$

$du = 11(2x^3-5)^{10} \cdot 6x^2$
$= 66x^2(2x^3-5)^{10}$

$dv = \frac{1}{2}(4x^2+5)^{-1/2} \cdot 8x$
$= 4x(4x^2+5)^{-1/2}$

$\frac{dy}{dx} = (2x^3-5)^{11} \cdot 4x(4x^2+5)^{-1/2} + (4x^2+5)^{1/2} \cdot 66x^2(2x^3-5)^{10}$

$\frac{dy}{dx} = \frac{2x(2x^3-5)^{10}}{\sqrt{4x^2+5}}\left[2(2x^3-5) + (4x^2+5) \cdot 33x\right]$

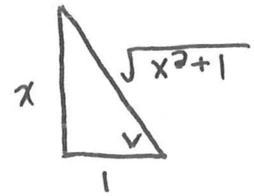
$= \frac{2x(2x^3-5)^{10}}{\sqrt{4x^2+5}}\left[4x^3-10 + 132x^3 + 165x\right]$

$= \frac{2x(2x^3-5)^{10}}{\sqrt{4x^2+5}}\left[136x^3 + 165x - 10\right]$

#12 $y = (x^2+1)^2 \cdot \text{TAN}^{-1}(x)$ LET $u = (x^2+1)^2$ $V = \text{TAN}^{-1}(x)$

$du = 2(x^2+1)\cdot 2x$ $\text{TAN } V = x$

$$y' = (x^2+1)^2 \cdot \frac{1}{(x^2+1)} + \text{TAN}^{-1}(x)\cdot 4x(x^2+1) \quad = 4x(x^2+1)$$



$$y' = x^2+1 + 4x(x^2+1)\,\text{TAN}^{-1}(x)$$

$$y' = (x^2+1)\left[1 + 4x \cdot \text{TAN}^{-1}(x)\right]$$

$$\text{SEC}^2(V) \cdot V' = 1$$

$$V' = \frac{1}{\text{SEC}^2 V}$$

$$V' = \cos^2 V$$

$$\boxed{V' = \frac{1}{x^2+1}}$$

#13 $y = \dfrac{e^x}{e^x - 1}$

$$\frac{dy}{dx} = \frac{(e^x-1)\cdot e^x - e^x(e^x)}{e^x - 1}$$: QUOTEENT RULE

$$= \frac{e^{2x} - e^x - e^{2x}}{[e^x-1]^2}$$

$$= \frac{-e^x}{[e^x-1]^2}$$

#14 $y = e^{3x} \cdot \ln(3x)$ $\qquad$ $u\,dv + v\,du$

$y' = e^{3x} \cdot \frac{1}{3x} \cdot 3 + \ln(3x) \cdot e^{3x} \cdot 3$

$y' = \frac{e^{3x}}{x} + 3\ln(3x) \cdot e^{3x}$

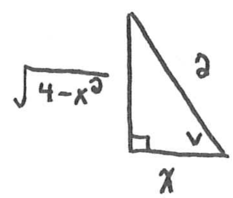
$y' = e^{3x}\left[\frac{1}{x} + 3\ln(3x)\right]$

#15 $y = \ln[\cos x] \cdot \sec^{-1}\left(\frac{2}{x}\right)$

LET $u = \ln[\cos x]$ $\qquad$ $V = \sec^{-1}\left(\frac{2}{x}\right)$

$du = \frac{1}{\cos x} \cdot -\sin x$ $\qquad$ $\sec V = \frac{2}{x}$

$\frac{dy}{dx} = \ln[\cos x] \cdot \frac{-1}{\sqrt{4-x^2}} + \sec^{-1}\left(\frac{2}{x}\right)(-\tan x)$ $\qquad$ $du = -\tan x$



$= \frac{-\ln[\cos x]}{\sqrt{4-x^2}} - \tan x \cdot \sec^{-1}\left(\frac{2}{x}\right)$

$\sec V \tan V \, dv = \frac{-2}{x^2}$

$dv = \cos V \cdot \cot V \cdot \frac{-2}{x^2}$

$= \frac{x}{2} \cdot \frac{x}{\sqrt{4-x^2}} \cdot \frac{-2}{x^2}$

$= \frac{-1}{\sqrt{4-x^2}}$

#16 $y = e^{\tan(x^2-1)} \cdot \cos(x^2-1)$

LET $u = e^{\tan(x^2-1)}$

$du = e^{\tan(x^2-1)} \cdot \sec^2(x^2-1) \cdot 2x$

$= 2x \cdot \sec^2(x^2-1) \cdot e^{\tan(x^2-1)}$

$$\frac{dy}{dx} = e^{\tan(x^2-1)} \cdot -2x\sin(x^2-1) + \cos(x^2-1) \cdot \underline{du}$$

LET $v = \cos(x^2-1)$

$dv = -\sin(x^2-1) \cdot 2x$

$= -2x\sin(x^2-1)$

$$= -2x \cdot \sin(x^2-1) \cdot e^{\tan(x^2-1)} + 2x \cdot \cos(x^2-1) \cdot \sec^2(x^2-1) \cdot e^{\tan(x^2-1)}$$

$$= -2x\sin(x^2-1) \cdot e^{\tan(x^2-1)} + 2x\sec(x^2-1) \cdot e^{\tan(x^2-1)}$$

$$= 2x\, e^{\tan(x^2-1)} \left[\sec(x^2-1) - \sin(x^2-1)\right]$$

#17 $y = \left[\csc\left(\ln(3x^2+2)\right)\right]^3$: POWER RULE FIRST

$$\frac{dy}{dx} = 3\left[\csc(\ln(3x^2+2))\right]^2 \cdot \underbrace{-\csc(\ln(3x^2+2)) \cdot \cot(\ln(3x^2+2))}_{\text{CHAIN FOR TRIG}} \cdot \underbrace{\frac{1}{3x^2+2}}_{\text{CHAIN FOR ANGLE W/ LN}} \cdot 6x$$

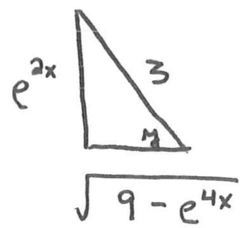
$$\frac{dy}{dx} \quad \frac{-18x}{3x^2+2}\csc^3(\ln(3x^2+2))\cot(\ln(3x^2+2))$$

#18

$$y = \sin^{-1}\left(\frac{e^{2x}}{3}\right)$$

$$\sin y = \frac{e^{2x}}{3}$$

CONSIDER:



$$\sin y = \frac{1}{3}e^{2x}$$

$$\cos y \cdot \frac{dy}{dx} = \frac{1}{3}e^{2x} \cdot 2$$

$$\frac{dy}{dx} = \frac{2}{3}e^{2x} \cdot \sec y$$

$$\frac{dy}{dx} = \frac{2}{3}e^{2x} \cdot \frac{3}{\sqrt{9-e^{4x}}}$$

$$= \frac{2}{\sqrt{9-e^{4x}}} \cdot e^{2x}$$