Power Rule: $$f(x) = x^a$$
$$f'(x) = a \cdot x^{a-1}$$

1. $$f(x) = x^2 + 3x + 5$$
$$f'(x) = 2x + 3$$

2. $$y = 5x^3 - 7x^2 + x - 4$$
$$y' = 15x^2 - 14x + 1$$

$$\frac{d}{dx}\left(\frac{-3}{x^4} + \frac{4}{x^2} - \frac{9}{x} + \frac{5}{8}\right)$$

$$\frac{d}{dx}\left(-3x^{-4} + 4x^{-2} - 9x^{-1} + \frac{5}{8}\right)$$

3. $$= 12x^{-5} - 8x^{-3} + 9x^{-2}$$

$$= \frac{12}{x^5} - \frac{8}{x^3} + \frac{9}{x^2}$$

$$= \frac{12 - 8x^2 + 9x^3}{x^5}$$

$$y = \frac{2}{x^3} - \frac{7}{x} + 3x - 11$$

$$y = 2x^{-3} - 7x^{-1} + 3x - 11$$

4. $D_x(y) = -6x^{-4} + 7x^{-2} + 3$

$$D_x(y) = \frac{-6}{x^4} + \frac{7}{x^2} + 3$$

$$D_x(y) = \frac{-6 + 7x^2 + 3x^4}{x^4}$$

$$f(x) = \frac{3}{\left(\sqrt{x-4}\right)^5}$$

$$f(x) = 3(x-4)^{-\frac{5}{2}}$$

5.

$$f'(x) = -\frac{15}{2}(x-4)^{-\frac{7}{2}}$$

$$f'(x) = -\frac{15}{2(x-4)^{\frac{7}{2}}}$$

$$y = \frac{3}{(x-7)^4}$$

6. $y = 3(x-7)^{-4}$

$$\frac{dy}{dx} = -12(x-7)^{-5}$$

$$\frac{dy}{dx} = \frac{-12}{(x-7)^5}$$

$$S(t) = 22t^2 - 5t + 9$$

7.

$$S'(t) = V(t) = 44t - 5$$

$$\frac{dy}{dx} = 9x^{\frac{2}{3}} - \frac{1}{2}x^{\frac{2}{5}} + 3x - \frac{11}{17}$$

8. $$\frac{d^2y}{dx^2} = 6x^{-\frac{1}{3}} - \frac{1}{5}x^{-\frac{3}{5}} + 3$$

$$\frac{d^2y}{dx^2} = \frac{6}{x^{\frac{1}{3}}} - \frac{1}{5x^{\frac{3}{5}}} + 3$$

$$g'(x) = 3x^4 - 2x^3 - x^2 + 11x - 3$$

9. $$\frac{d^2y}{dx^2} = 12x^3 - 6x^2 - 2x + 11$$

$$\frac{d^3y}{dx^3} = 36x^2 - 12x - 2$$

$$f(x) = u \cdot v$$

Product Rule:

$$f'(x) = u \cdot dv + v \cdot du$$

1. $f(x) = 13x^2(x-2)^5$

Let $u = 13x^2$ and $v = (x-2)^5$
So $du = 26x$ and $dv = 5(x-2)^4$

$$f'(x) = u \cdot dv + v \cdot du$$

$$f'(x) = 13x^2 \cdot 5(x-2)^4 + (x-2)^5 \cdot 26x$$

$$f'(x) = 13x(x-2)^4[5x + 2(x-2)]$$

$$f'(x) = 13x(x-2)^4[7x-4]$$

2. $y = 3\pi x \cdot \sin x$

Let $u = 3\pi x$ and $v = \sin x$
So $du = 3\pi$ and $dv = \cos x$

$$y' = u \cdot dv + v \cdot du$$

$$y' = 3\pi x \cdot \cos x + \sin x \cdot 3\pi$$

$$y' = 3\pi(x \cdot \cos x + \sin x)$$

3. $\frac{d}{dt}\left((t-4)^3 \cdot (t+9)^7\right)$

Let $u = (t-4)^3$ and $v = (t+9)^7$
So $du = 3(t-4)^2$ and $dv = 7(t+9)^6$

$$\frac{dy}{dt} = u \cdot dv + v \cdot du$$

$$\frac{dy}{dt} = (t-4)^3 \cdot 7(t+9)^6 + (t+9)^7 \cdot 3(t-4)^2$$

$$\frac{dy}{dt} = (t-4)^2 (t+9)^6 \left[7 \cdot (t-4) + 3(t+9)\right]$$

$$\frac{dy}{dt} = (t-4)^2 (t+9)^6 \left[7t - 28 + 3t + 27\right]$$

$$\frac{dy}{dt} = (t-4)^2 (t+9)^6 \left[10t - 1\right]$$

4. $y = (z+8)^5 \cdot (z-11)^3 \cdot (z+3)^7$

$$D_z(y) = 5(z+8)^4 \cdot (z-11)^3 \cdot (z+3)^7 + (z+8)^5 \cdot 3(z-11)^2 \cdot (z+3)^7 + (z+8)^5 \cdot (z-11)^3 \cdot 7(z+3)^6$$

$$D_z(y) = (z+8)^4 \cdot (z-11)^2 \cdot (z+3)^6 \cdot \left[5(z-11)(z+3) + 3(z+8)(z+3) + 7(z+8)(z-11)\right]$$

$$D_z(y) = (z+8)^4 \cdot (z-11)^2 \cdot (z+3)^6 \cdot \left[5(z^2 - 8z - 33) + 3(z^2 + 11z + 24) + 7(z^2 - 3z - 88)\right]$$

$$D_z(y) = (z+8)^4 \cdot (z-11)^2 \cdot (z+3)^6 \cdot \left[5z^2 - 40z - 165 + 3z^2 + 33z + 72 + 7z^2 - 21z - 616\right]$$

$$D_z(y) = (z+8)^4 \cdot (z-11)^2 \cdot (z+3)^6 \cdot \left[15z^2 - 28z - 709\right]$$

5. $y = \sin w \cdot \tan w$

$$\frac{dy}{dw} = \sin(w) \cdot \underline{\sec^2(w)} + \tan(w)\underline{(-\cos(w))}$$

$$\frac{dy}{dw} = \sin(w) \cdot \sec^2(w) + \frac{\sin(w)}{\cos(w)}(-\cos(w))$$

$$\frac{dy}{dw} = \sin(w)(\sec^2(w) - 1)$$

$$\frac{dy}{dw} = \sin(w)(\tan^2(w))$$

6. $S(t) = 5t^2(\tan t) - 3t(\sec t) - 11$

$$S'(t) = 5t^2 \cdot \underline{\sec^2 t} + \tan t \cdot 10t - 3t\,\underline{\sec t \tan t} + \sec t\,\underline{(-3)}$$

$$V(t) = 5t\left[t\sec^2 t + 2\tan t\right] - 3\sec t\left[t\tan t + 1\right]$$

7. $\frac{dy}{dx} = 9x^{\frac{2}{3}}(x-3)^{\frac{1}{2}}$

$$\frac{dy}{dx} = 9x^{\frac{2}{3}}\underline{\frac{1}{2}(x-3)^{-\frac{1}{2}}} + (x-3)^{\frac{1}{2}} \cdot \underline{6x^{-\frac{1}{3}}}$$

$$\frac{dy}{dx} = \frac{9x^{\frac{2}{3}}}{2(x-3)^{\frac{1}{2}}} + \frac{6(x-3)^{\frac{1}{2}}}{x^{\frac{1}{3}}}$$

$$\frac{dy}{dx} = \frac{9x^{\frac{2}{3}}[x^{\frac{1}{3}}]}{2(x-3)^{\frac{1}{2}}[x^{\frac{1}{3}}]} + \frac{6(x-3)^{\frac{1}{2}}[2\cdot(x-3)^{\frac{1}{2}}]}{x^{\frac{1}{3}}[2\cdot(x-3)^{\frac{1}{2}}]}$$

$$\frac{dy}{dx} = \frac{9x + 12(x-3)}{2(x-3)^{\frac{1}{2}}[x^{\frac{1}{3}}]}$$

$$\frac{dy}{dx} = \frac{21x - 36}{2(x-3)^{\frac{1}{2}}[x^{\frac{1}{3}}]}$$

8. $h''(t) = (t-3)^4 \cdot (t+1)^2 \cdot (5t-4)$

$$h''' = \underline{4(t-3)^3} \cdot (t+1)^2 \cdot (5t-4) + (t-3)^4 \cdot \underline{2(t+1)} \cdot (5t-4) + (t-3)^4 \cdot (t+1)^2 \cdot \underline{5}$$

$$h''' = (t-3)^3 \cdot (t+1) \cdot \left[4\cdot(t+1)(5t-4) + 2(t-3)(5t-4) + 5(t-3)(t+1)\right]$$

$$h''' = (t-3)^3 \cdot (t+1) \cdot \left[4(5t^2 + t - 4) + 2(5t^2 - 19t + 12) + 5(t^2 - 2t - 3)\right]$$

$$h''' = (t-3)^3 \cdot (t+1) \cdot \left[20t^2 + 4t - 16 + 10t^2 - 38t + 24 + 5t^2 - 10t - 15\right]$$

$$h''' = (t-3)^3 \cdot (t+1) \cdot \left[35t^2 - 44t - 7\right]$$

$$f(x) = \frac{High}{Low}$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{High}{Low}\right] = \frac{Low \cdot d(High) - High \cdot d(Low)}{Low^2}$$

1. $g(\lambda) = \dfrac{\lambda}{1 - \sin\lambda}$

$$g'(\lambda) = \frac{(1 - \sin\lambda) \cdot 1 - \lambda(-\cos\lambda)}{(1 - \sin\lambda)^2}$$

$$g'(\lambda) = \frac{1 - \sin\lambda + \lambda \cdot \cos\lambda}{(1 - \sin\lambda)^2}$$

2. $y = \dfrac{(x-3)^5}{(x+4)^2}$

$$y' = \frac{(x+4)^2 \cdot 5(x-3)^4 - (x-3)^5 \cdot 2(x+4)}{[(x+4)^2]^2}$$

$$y' = \frac{(x+4)(x-3)^4[5(x+4) - 2(x-3)]}{(x+4)^4}$$

$$y' = \frac{(x-3)^4[5x + 20 - 2x + 6]}{(x+4)^3}$$

$$y' = \frac{(x-3)^4[3x + 26]}{(x+4)^3}$$

3. $\frac{d}{dx}(\tan(x))$

$$f(x) = \frac{\sin x}{\cos x}$$

$$f'(x) = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$f'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$f'(x) = \frac{1}{\cos^2 x}$$

$$f'(x) = \sec^2 x$$

4. $y = \left(\frac{3h-2}{2h+3}\right)\cdot (5h+1)$

$$D_h(y) = \left(\frac{3h-2}{2h+3}\right)\cdot [5] + (5h+1)\left[\frac{(2h+3)\cdot [3] - (3h-2)\cdot [2]}{(2h+3)^2}\right]$$

$$D_h(y) = \left(\frac{3h-2}{2h+3}\right)\cdot [5] + (5h+1)\left[\frac{6h+9-6h+4}{(2h+3)^2}\right]$$

$$D_h(y) = \left(\frac{15h-10}{2h+3}\right) + \left[\frac{13(5h+1)}{(2h+3)^2}\right]$$

$$D_h(y) = \left(\frac{15h-10}{2h+3}\right)\cdot \frac{2h+3}{2h+3} + \left[\frac{13(5h+1)}{(2h+3)^2}\right]$$

$$D_h(y) = \frac{30h^2+25h-30}{(2h+3)^2} + \frac{65h+13}{(2h+3)^2}$$

$$D_h(y) = \frac{30h^2+90h-17}{(2h+3)^2}$$

5. $y = \dfrac{\sin x}{1-\cos x}$

$$\frac{dy}{dx} = \frac{(1-\cos x)(\cos x) - \sin x(\sin x)}{(1-\cos x)^2}$$

$$\frac{dy}{dx} = \frac{\cos x - \cos^2 x - \sin^2 x}{(1-\cos x)^2}$$

$$\frac{dy}{dx} = \frac{\cos x - (\cos^2 x + \sin^2 x)}{(1-\cos x)^2}$$

$$\frac{dy}{dx} = \frac{\cos x - 1}{(1-\cos x)^2}$$

$$\frac{dy}{dx} = \frac{-1}{1-\cos x} = \frac{1}{\cos x - 1}$$

6. $S(t) = \dfrac{t^2 - 3t + 5}{t+1} + 5t(t+1)^3 + 9$

$$S'(t) = \frac{(t+1)(2t-3) - (t^2 - 3t + 5)\cdot 1}{(t+1)^2}$$
$$+5t\cdot 3(t+1)^2 + (t+1)^3\cdot 5$$

$$V(t) = \frac{2t^2 - t - 3 - t^2 + 3t - 5}{(t+1)^2}$$
$$+5(t+1)^2\cdot [3t + t + 1]$$

$$V(t) = \frac{t^2 + 2t - 8}{(t+1)^2} + 5(t+1)^2\cdot [4t+1]$$

7. $\frac{dy}{dx} = 3x^2\left(4 - \frac{1}{x-2}\right)$

$$\frac{dy}{dx} = 3x^2\left(4 - (x-2)^{-1}\right)$$

$$\frac{d^2y}{dx^2} = 3x^2 \cdot \underline{(x-2)^{-2}} + \left(4 - (x-2)^{-1}\right) \cdot \underline{6x}$$

$$\frac{d^2y}{dx^2} = \frac{3x^2}{(x-2)^2} + 24x - \frac{6x}{(x-2)}$$

$$\frac{d^2y}{dx^2} = \frac{3x^2}{(x-2)^2} + \frac{24x[x^2 - 4x + 4]}{[x-2]^2} - \frac{6x[x-2]}{(x-2)[x-2]}$$

$$\frac{d^2y}{dx^2} = \frac{3x^2 + 24x^3 - 96x^2 + 96x - 6x^2 + 12x}{(x-2)^2}$$

$$\frac{d^2y}{dx^2} = \frac{24x^3 - 99x^2 + 108x}{(x-2)^2}$$

Chain Rule:

$g(\mu) = 5\tan(3\mu^2)$

1. $g'(\mu) = 5\sec^2(3\mu^2)\cdot 6\mu$

$g'(\mu) = 30\mu\sec^2(3\mu^2)$

$y = \sin^5(x^4)$

$y = [\sin(x^4)]^5$

2.

$y' = 5\cdot [\sin(x^4)]^4\left[\cos(x^4)\cdot 4x^3\right]$

$y' = 20x^3\cdot \sin^4(x^4)\cdot \cos(x^4)$

$\frac{d}{dx}\left((3x^5 - 8)^{11}\right) = 11(3x^5 - 8)^{10}\cdot 15x^4$

3.

$\frac{d}{dx}\left((3x^5 - 8)^{11}\right) = 165x^4\cdot (3x^5 - 8)^{10}$

$y = 11\cos^5(4x^2 - 5)^3$

$y = 11[\cos(4x^2 - 5)^3]^5$

4.

$\frac{dy}{dx} = 55[\cos(4x^2 - 5)^3]^4\cdot \left(-\sin(4x^2 - 5)^3\right)\cdot 3(4x^2 - 5)^2\cdot 8x$

$\frac{dy}{dx} = -1320x\cdot (4x^2 - 5)^2\cos^4(4x^2 - 5)^3\cdot \sin(4x^2 - 5)^3$

5. $\frac{d}{dx}\left((5x^3+2)^{10}\cdot(4x^3-2)^7\right)$

$u=(5x^3+2)^{10}$
$du=10\cdot(5x^3+2)^9\cdot 15x^2$
$du=150x^2\cdot(5x^3+2)^9$

$v=(4x^3-2)^7$
$dv=7(4x^3-2)^6\cdot 12x^2$
$dv=84x^2(4x^3-2)^6$

$$\frac{d}{dx}\left((5x^3+2)^{10}\cdot(4x^3-2)^7\right)=u\cdot dv+v\cdot du$$

$$=(5x^3+2)^{10}\cdot 84x^2(4x^3-2)^6+(4x^3-2)^7\cdot 150x^2\cdot(5x^3+2)^9$$

$$=6x^2(4x^3-2)^6\cdot(5x^3+2)^9[14(5x^3+2)+25(4x^3-2)]$$

$$=6x^2(4x^3-2)^6\cdot(5x^3+2)^9[170x^3-22]$$

6. $S(t) = (4t^3 - 5)^{11} \cdot (5t^2 - 7)^9$

$u = (4t^3 - 5)^{11}$

$u' = 11 \cdot (4t^3 - 5)^{10} \cdot 12t^2$

$u' = 132t^2 \cdot (4t^3 - 5)^{10}$

$v = (5t^2 - 7)^9$

$v' = 9 \cdot (5t^2 - 7)^8 \cdot 10t$

$v' = 90t \cdot (5t^2 - 7)^8$

$S'(t) = u \cdot v' + v \cdot u'$

$= (4t^3 - 5)^{11} \cdot 90t \cdot (5t^2 - 7)^8 + (5t^2 - 7)^9 \cdot 132t^2 \cdot (4t^3 - 5)^{10}$

$= 6t(5t^2 - 7)^8 \cdot (4t^3 - 5)^{10}[15(4t^3 - 5) + 22t(5t^2 - 7)]$

$= 6t(5t^2 - 7)^8 \cdot (4t^3 - 5)^{10}[170t^3 - 154t - 75]$

7. $\frac{dy}{dx} = 2\sin^2(7x^3)\cdot \cos^4(5x^5)$

$u = 2\sin^2(7x^3)$

$u = 2\left[\sin(7x^3)\right]^2$

$du = 4\left[\sin(7x^3)\right]^1 \cdot \cos(7x^3)\cdot 21x^2$

$du = 84x^2\left[\sin(7x^3)\right]\cdot \cos(7x^3)$

$v = \cos^4(5x^5)$

$v = \left[\cos(5x^5)\right]^4$

$dv = 4\left[\cos(5x^5)\right]^3 \cdot \left(-\sin(5x^5)\right)\cdot 25x^4$

$dv = -100x^4\left[\cos(5x^5)\right]^3 \cdot \left(\sin(5x^5)\right)$

$\frac{d^2y}{dx^2} = u\cdot dv + v\cdot du$

$\frac{d^2y}{dx^2} = 2\sin^2(7x^3)\cdot -100x^4\left[\cos(5x^5)\right]^3 \cdot \left(\sin(5x^5)\right)$
$+\cos^4(5x^5)\cdot 84x^2\left[\sin(7x^3)\right]\cdot \cos(7x^3)$

$\frac{d^2y}{dx^2} = -200x^4 \cdot \sin^2(7x^3)\cdot \cos^3(5x^5)\cdot \left(\sin(5x^5)\right)$
$+84x^2\cdot \cos^4(5x^5)\cdot \sin(7x^3)\cdot \cos(7x^3)$

8. $y = \left(\dfrac{\sin(5h)}{\cos(7h)}\right)$

$$\frac{dy}{dx} = \frac{\cos(7h)\cdot \cos(5h)\cdot 5 - \sin(5h)\cdot(-\sin(7h))\cdot 7}{[\cos(7h)]^2}$$

$$\frac{dy}{dx} = \frac{5\cos(7h)\cdot \cos(5h) + 7\sin(5h)\cdot \sin(7h)}{[\cos(7h)]^2}$$

9. $g(\alpha) = \dfrac{(3\alpha^4 - 5)^5}{(2\alpha^5 - 4)^3}$

$$g'(\alpha) = \frac{(2\alpha^5 - 4)^3\cdot 5(3\alpha^4 - 5)^4\cdot 12\alpha^3 - (3\alpha^4 - 5)^5\cdot 3(2\alpha^5 - 4)^2\cdot 10\alpha^4}{\left[(2\alpha^5 - 4)^3\right]^2}$$

$$g'(\alpha) = \frac{60\alpha^3(2\alpha^5 - 4)^3(3\alpha^4 - 5)^4 - 30\alpha^4(3\alpha^4 - 5)^5(2\alpha^5 - 4)^2}{(2\alpha^5 - 4)^6}$$

$$g'(\alpha) = \frac{60\alpha^3(2\alpha^5 - 4)^3(3\alpha^4 - 5)^4 - 30\alpha^4(3\alpha^4 - 5)^5(2\alpha^5 - 4)^2}{(2\alpha^5 - 4)^6}$$

$$g'(\alpha) = \frac{30\alpha^3(2\alpha^5 - 4)^2(3\alpha^4 - 5)^4\left[2(2\alpha^5 - 4) - \alpha(3\alpha^4 - 5)\right]}{(2\alpha^5 - 4)^6}$$

$$g'(\alpha) = \frac{30\alpha^3(3\alpha^4 - 5)^4(\alpha^5 + 5\alpha - 8)}{(2\alpha^5 - 4)^4}$$

10. $y = \left(\dfrac{\sin^2(5h^3 - 2)}{\cos^2(7h^2 + 3)}\right)$

$$D_h(y) = \frac{\cos^2(7h^2+3)\cdot 2\left[\sin(5h^3-2)\right]^1 \cdot \cos(5h^3-2)\cdot 15h^2 - \sin^2(5h^3-2)\cdot 2\left[\cos(7h^2+3)\right]^1 \cdot (-\sin(7h^2+3))\cdot 14h}{\left[\cos^2(7h^2+3)\right]^2}$$

$$D_h(y) = \frac{2h\sin(5h^3-2)\cos(7h^2+3)\left[15h\cos(7h^2+3)\cos(5h^3-2)\cdot +14\sin(5h^3-2)\sin(7h^2+3)\right]}{\cos^4(7h^2+3)}$$

$$D_h(y) = \frac{2h\sin(5h^3-2)\left[15h\cos(7h^2+3)\cos(5h^3-2)\cdot +14\sin(5h^3-2)\sin(7h^2+3)\right]}{\cos^3(7h^2+3)}$$