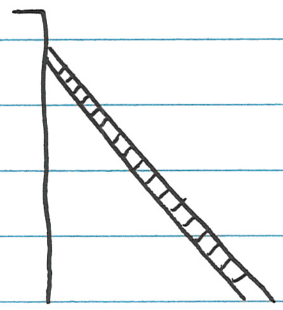
9



3 ft/sec →

* AS THE LADDER SLIDES DOWN THE WALL, THERE WILL BE A SERIES OF SNAP SHOTS IN WHICH THERE ALWAYS EXIST A RIGHT TRIANGLE.

SO, $x^2+y^2=22^2$

OBJECTIVE: FIND dy/dt WHEN $x = 7$ ft

NEED y-VALUE FOR WHEN $x=7$

$$x^2+y^2=22^2$$
$$7^2+y^2=22^2$$
$$y=\sqrt{435}$$

TAKE IMPLICIT DERIVATIVE, THEN REPRESENT RESULT WITH RESPECT TO TIME.

$$2x\,dx + 2y\,dy = 0$$

$\frac{dx}{dt} = 3$ WHEN $x=7$ AND $y=\sqrt{435}$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

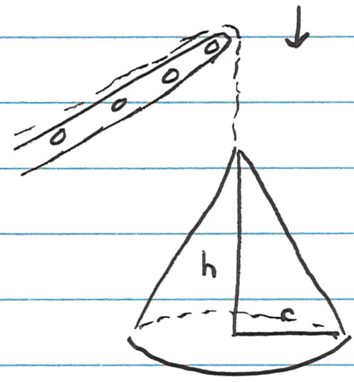
PLUG IN KNOWN RESULTS, SOLVE FOR UNKNOWN

$$2(7)\cdot 3 + 2(\sqrt{435})\frac{dy}{dt} = 0$$
$$2\sqrt{435}\,\frac{dy}{dt} = -42$$
$$\frac{dy}{dt} = \frac{-42}{2\sqrt{435}} = -1.0069$$

$\frac{dy}{dt}$ IS <u>DECREASING</u> AT A RATE OF 1.0069 ft/sec WHEN THE FOOT OF THE LADDER IS 7 ft OUT FROM THE WALL.

#10

$18\ ft^3/\text{MIN}$



$h = 4\tfrac{1}{2} r$

$h = \frac{9}{2} r \quad \Rightarrow \quad \frac{2}{9} h = r$

$$\frac{dV}{dt} = 18\ ft^3/\text{MIN}$$

FORMULA FOR CONE: $V = \frac{1}{3}\pi r^2 h$ FIND $\frac{dh}{dt}$ WHEN $h = 10$

CONVERT FORMULA TO ONLY REPRESENT VOLUME & HEIGHT

$$V = \frac{1}{3}\pi\left[\frac{2}{9}h\right]^2 h$$

$$V = \frac{4}{243}\pi h^3$$

TAKE IMPLICIT DERIVATIVE, THEN REPRESENT WITH RESPECT TO TIME

$$dV = \frac{4}{81}\pi h^2\, dh$$

$$\frac{dV}{dt} = \frac{4}{81}\pi h^2 \frac{dh}{dt}$$

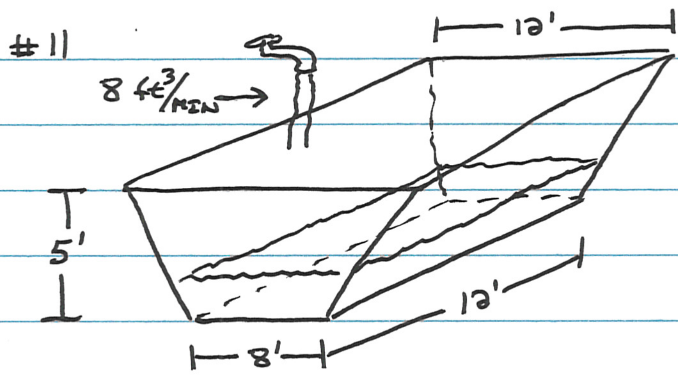
PLUG IN KNOWN VALUES, SOLVE FOR UNKNOWN

$$18 = \frac{4}{81}\pi (10)^2 \frac{dh}{dt}$$

$$1.1602 = \frac{dh}{dt}$$

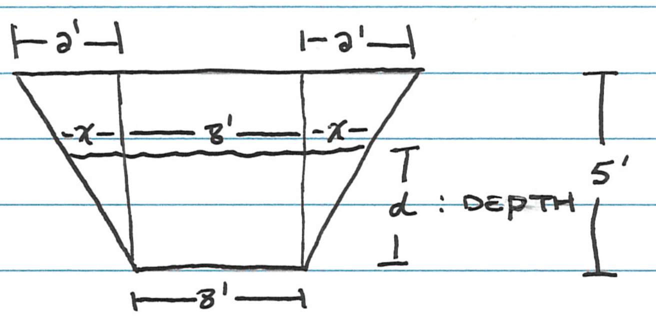
$\frac{dh}{dt}$ IS <u>INCREASING</u> AT A RATE $1.1602\ ft/\text{MIN}$ WHEN THE HEIGHT OF THE PILE IS 10 ft.

#11



HOW FAST IS WATER LEVEL RISING WHEN WATER IS 3.4 FEET DEEP?

*NOTICE THAT AS THE TROUGH FILLS UP, IT IS NOT EXACTLY A SET OF SIMILAR TRAPEZOIDS, BUT THERE DOES EXIST SIMILAR TRIANGLES WITHIN THE TRAPEZOID.



$$\frac{d}{5} = \frac{x}{2}$$

$$\frac{2d}{5} = x$$

VOLUME FOR TROUGH: $V = \frac{1}{2}(b_1 + b_2)\, h \cdot l$

b_1: IS CONSTANT AT 8'

b_2: $8 + 2x$ } FOR ANY MOMENT IN TIME AS TROUGH FILLS

h: d

l: IS CONSTANT AT 12'

SO, $V = \frac{1}{2}\left[8 + 8 + 2x\right] d \cdot 12$

OBJECTIVE: FIND RATE WATER LEVEL CHANGING $\left(\frac{dd}{dt}\right)$ WHEN $d = 3.4$

REPLACE x WITH EXPRESSION IN TERMS OF d AND SIMPLIFY

$$V = \frac{1}{2}\left[8 + 8 + 2\left[\frac{2}{5}d\right]\right] d \cdot 12$$

$$V = 6d\left(16 + \frac{4}{5}d\right)$$

$$V = 96d + \frac{24}{5}d^2$$

$$\frac{dV}{dt} = 96\frac{dd}{dt} + \frac{48}{5}d\,\frac{dd}{dt}$$: IMPLICIT DERIVATIVE

$$8 = \left[96 + \frac{48}{5}(3.4)\right]\frac{dd}{dt}$$: PLUG IN $\frac{dV}{dt}$ AND d

$$.0622 \text{ ft/min} = \frac{dd}{dt}$$

THE DEPTH IS INCREASING AT A RATE .0622 ft/min WHEN THE WATER LEVEL REACHES A DEPTH OF 3.4 FEET.