

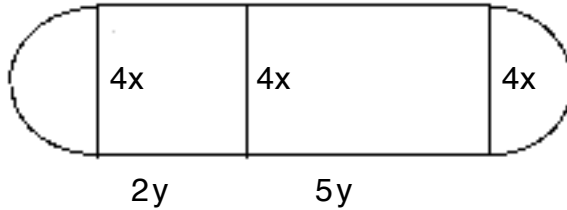
**Test: Applications of Derivatives**

Name \_\_\_\_\_

Complete each section,

**Have all work for each problem on one page, and only one problem per page.****Optimization (one problem per page)**

1. A toad farmer wants to use 840 m of fence to construct a pen that looks as follows. What is the maximum area she can enclose.



2. You must construct a closed rectangular box with volume  $1225 \text{ in}^3$  and a bottom that is twice as long as it is wide. Find the dimensions of the box that will minimize the total surface area.
3. A rectangular box with no top is to be constructed by cutting equal squares from each corner of a  $12 \times 9$  ft rectangular piece of material and bending the sides up. Express the volume of the box as a **function of its height**. Find the dimensions of the box that has maximum volume.
4. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost  $\$10 / \text{ft}^2$  and the material used to build the sides cost  $\$6 / \text{ft}^2$ . If the box must have a volume of  $50 \text{ ft}^3$  determine the dimensions that will **minimize the cost** to build the box.
5. A manufacturer needs to make a cylindrical can that will hold 3.5 liters of liquid. Determine the dimensions of the can that will **minimize the amount of material used (surface area)** in its construction.
6. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions of a Norman window with maximum area if the total exterior perimeter is 52 feet.
7. Find the points  $(x, y)$  on the graph of  $y = x^2 - 6x + 11$  that are closest to the point  $(3, 7)$ . (Answer will not be an integer for neither  $x$  nor  $y$ )
8. A right circular **cylinder** is **inscribed** in a right circular **cone** of radius 9 cm and a height of 15 cm. Find the dimensions of the cylinder with maximum volume. (look to compare cones somewhere in this problem)

**Related Rates: (one problem per page)**

9. One end of a 22 ft ladder is on the ground, and the other rests on a vertical wall if the bottom is pulled away from the wall at a rate of 3 ft / sec, how fast is the top of the ladder sliding down the wall when the foot of the ladder is 7 ft from the wall?

10. Sand falls from a conveyer belt into a conical pile with a rate of  $18 \text{ ft}^3 / \text{min}$ . The height of the pile is always four and a half times the radius. How fast is the altitude of the pile increasing when the pile is 10 feet high?

11. The cross section of a 12-foot trough is an isosceles trapezoid with a 8-foot lower base and a 12-foot upper base, and an altitude of 5 feet. Water is running into the trough at a rate of 8 cubic feet per minute. How fast is the water level rising when the water is 3.4 feet deep?

**Using Calculator to Find All Real Roots (any calculator method)**

**(Problems 11-14 should all be put on one page)**

Find the roots to the following (Decimals should be taken out to **four places**)

12.  $f(x) = x^4 - x^3 - 75$

13.  $f(x) = x^6 + 7x^2 - 4$

14.  $y = x^3 + 3x - 5x\sqrt{x+3} + 4$

15.  $y = x^4 + 7x^3 - 13x\sqrt{x+2} - 4$

**L'Hopital's Rule (Do All Six) 15-20 on one page**

16.  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)$

17.  $\lim_{x \rightarrow 1} \left( \frac{x^3 - 1}{4x^3 - x - 3} \right)$

18.  $\lim_{x \rightarrow 2} \left( \frac{\sqrt{x^2 + 5} - 3}{x^2 - 4} \right)$

19.  $\lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\sin x - \cos x}{x - \frac{\pi}{4}} \right)$

20.  $\lim_{x \rightarrow 1} \left( \frac{x^3 - 1}{x^2 - 1} \right)$

21.  $\lim_{x \rightarrow 2} \left( \frac{x^3 - 8}{x^4 - 16} \right)$

**More Fun Problems (one per page)**

22. Two poles are 24 feet apart. One is 9 ft tall while the other is 20 ft tall. If a wire is to be attached to the top of each find the **minimum** length of wire needed if the wire must touch the ground between the poles for a length of 2.5 feet. In other words, it must have a 2.5 foot section that actually sits on the ground.

23. Find the point(s) where the instantaneous velocity is equal to the average rate of change for the following function.

Part A.  $S(t) = 6t^3 - 4t^2 - 9t + 2$  from  $t = -2.45$  to  $t = 3.61$

Part B.  $M(x) = 3x^4 - 9x^3 - 4x^2 + 2x + 8$  for the interval  $[-2.23, .42]$

24. Suppose that we know that  $F(x)$  is continuous and differentiable on  $[3, 17]$ . It is known that  $F(3) = -4$ , and  $3 \leq f'(x) \leq 7 \quad \forall x \in [3, 17]$ . Find the least and greatest value possible for  $F(17)$ .

25. An apartment complex has 250 apartments to rent. If they rent  $x$  apartments then their monthly profit, in dollars, is given by,  $P(x) = -8x^2 + 3200x - 80,000$  What is the optimal number of apartments to rent in order to maximize profit?

26. A production facility is capable of producing 60,000 widgets in a day and the total daily cost of producing  $x$  widgets in a day is given by,  $c(x) = 250,000 + .08x + \frac{200,000,000}{x}$

How many widgets per day should they produce in order to minimize production costs?

27. There was once a sand filled sphere with a radius of 42 feet that was punctured in such a way that  $1/72$  of its volume leaked out every second. The sand was being captured in a conical container with a radius of 35 ft and a height of 90 ft. How was the radius of the cone **increasing** when it reached a depth of 55 ft?

1.

An empty rectangular box with a black border, intended for a response to question 1.

2.

An empty rectangular box with a black border, intended for a response to question 2.

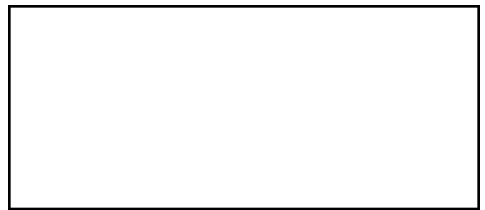
3.

An empty rectangular box with a black border, intended for a response to question 3.

4.

An empty rectangular box with a black border, intended for a response to question 4.

5.

An empty rectangular box with a black border, intended for a response to question 5.

6.

An empty rectangular box with a black border, intended for a response to question 6.

7.



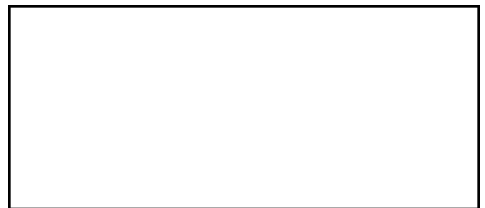
8.



9.



10.



11.



12.



13.



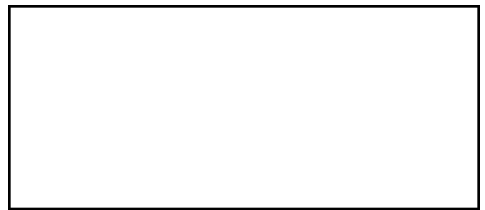
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15.



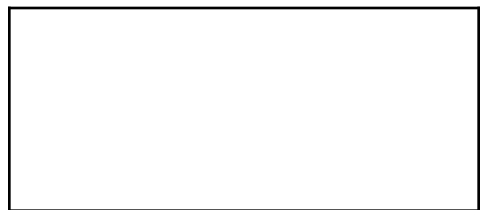
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17.



18.



19.



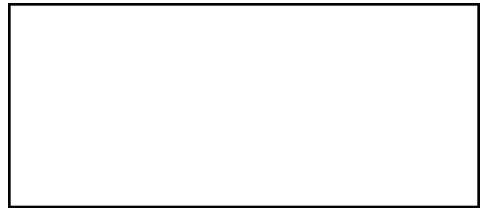
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21.



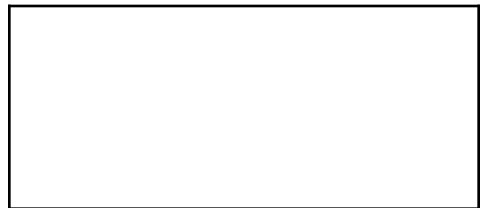
22.



23.



24.



25.



26.



27.

