Age Word Problems

Notes regarding age word problems

The structure for these word problems is consistent. The first sentence will provide information about what names to write down as well as an algebraic relationship between or among those names.

The second sentence will be broken down into two parts. At the beginning of the second sentence there will be an introductory phrase such as "Five years ago." This will be used to create a second column where each of the preceding algebraic expressions are changed by either adding or subtracting the indicated amount. "five years ago" would imply that each expression needs to be reduced by subtracting five. Everything after the introductory phrase in the second sentence will translate into an equation. That equation must be constructed using the second column of algebraic expressions.

1. Larry is 15 years older than Beth. In 4 years, Larry will be one year older than twice Beth's age. Find each age today.

Names	Ages	In four years (+ 4)
Larry	B + 15	B + 19
Beth	B	B + 4

Larry = 2(Beth) + 1

$B + 19 = 2(B + 4) + 1$

$B + 19 = 2B + 8 + 1$

$B + 19 = 2B + 9$

$19 - 9 = 2B - B$

$10 = B$

Larry is 25 years old

Beth is 10 years old

2. Michelle is 2 years older than Judy, and Terry is 3 times as old as Michelle. In 7 years, Terry's age will be 5 more than the sum of the other two ages. Find the age of each now.

Note: A common practice is to declare the variable for the item referred to most often. In this example it would follow to declare M for Michelle.

Names	Ages	In seven years (+ 7)
Michelle	M	M + 7
Judy	M – 2	M + 5
Terry	3M	3M + 7

Terry = (Michelle + Judy) + 5

$$3m + 7 = [(m + 7) + (m + 5)] + 5$$

$$3m + 7 = 2m + 12 + 5$$

$$3m + 7 = 2m + 17$$

$$3m - 2m = 17 - 7$$

$$m = 10$$

Michelle is 10 years old

Judy is 8 years old

Terry is 30 years old

Integer Problems

Notes regarding integer word problems

Although there are several methods for declaring variables for such problems, the method discussed in class makes every attempt to take advantage of given information. For example, all even integers are represented as a multiple of two and then it should follow that odd integers could be found by simply adding one. In some cases not enough information is given to declare for even or odd but it is known that the numbers will alternate back and forth. In those circumstances using the more general case of x will suit better.

Consecutive Even Integers		*Consecutive Odd Integers*		*Consecutive Integers*	
First:	*2n*	*First:*	*2n + 1*	*First:*	*x*
Second:	*2n + 2*	*Second:*	*2n + 3*	*Second:*	*x + 1*
Third:	*2n + 4*	*Third:*	*2n + 5*	*Third:*	*x + 2*

3. Find three consecutive **even integers** such that twice the least increased by three times the greatest is 102. Find the integers.

First:	2n
Second:	2n + 2
Third:	2n + 4

2 (First) + 3 (Third) = 102

$2(2n) + 3(2n + 4) = 102$

$4n + 6n + 12 = 102$

$10n + 12 = 102$

$10n = 102 - 12$

$10n = 90$

$n = 9$

First:	2n	= 18
Second:	2n + 2	= 20
Third:	2n + 4	= *22*

4. Five times the smallest of three consecutive **odd integers** is ten more than twice the largest. Find the integers.

First: 2n + 1
Second: 2n + 3
Third: 2n + 5

5 (First) = 2 (Third) +10

$$5(2n+1) = 2(2n+5)+10$$

$$10n+5 = 4n+10+10$$

$$10n+5 = 4n+20$$

$$10n-4n = 20-5$$

$$6n = 15$$

$$n = 2\frac{1}{2}$$

Note: that the solution to the equation is not an integer. this should be a hint that there is no solution. It comes directly from the method chosen for declaring variables. Technically if one were to plug in 2.5 for n there are integer answers of 6 , 8, and 10 respectively but the definitely are not odd integers.

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Number Problems

Notes regarding number word problems

There are three key methods for declaring variables under this section. The first item to consider for each problem will usually be found in the last sentence. The reader needs to decide how many numbers need to be found. If it is a single number problem then declare a single variable and begin to write the equation immediately. If the problem asks for more than one number then the set up will follow one of two avenues. The first will be for "the sum of two numbers ..." while the second will be " the difference of two numbers..."

Example for Sum:
The sum of two numbers is 45.
One: X
Other: $45 - X$

Example for Difference:
The difference of two numbers is 15.
One: $X + 15$
Other: X

Notice that "sum" will use a subtraction sign and the X's will cancel out when the expressions are combined while the "difference" will use an addition sign and that the X's are canceled out through the operation of subtraction. In other words, the declared expressions should include the opposite signs to the operation indicated.

5. The sum of two numbers is 17. Three times one number is one more than twice the other.

One: x
Other: 17 – x

3 (One) = 2 (Other) + 1

$$3(x) = 2(17 - x) + 1$$
$$3x = 34 - 2x + 1$$
$$3x = 35 - 2x$$
$$3x + 2x = 35$$
$$5x = 35$$
$$x = 7$$

One:	x = 7
Other:	17 – x = 10

6. The difference of two numbers is 11. Twice one number is six more than the other.

One: x + 11
Other: x

2 (One) = (Other) + 6

$$2(x+11)=(x)+6$$
$$2x+22=x+6$$
$$2x-x=6-22$$
$$x=-16$$

One:	x + 11 = –5
Other:	x = – 16

Note: If one were following the method taught in class then answers will match. Examples like problem 5 will always produce the same set of numbers when expressions are swapped. However, problem six will lead to a different set of answers if the expressions are swapped. Those answers are equally correct but not pursued for the sake of simplicity.

6.1
One: x + 11
Other: x

2 (Other) = (One) + 6

$$2(x)=(x+11)+6$$
$$2x=x+17$$
$$2x-x=17$$
$$x=17$$

One:	x + 11 = 28
Other:	x = 17

7. Twice a number increased by twelve, is thirty five less than three times the number. Find the number.
Note: The question only wants a single value. So declare one variable, and write the equation.

Number: w

$$2w + 12 = 3w - 35$$

$$12 + 35 = 3w - 2w$$

$$47 = w$$

Number: w = 47

Mixture Problems

Notes regarding mixture word problems

There are three column headings that should be remembered when completing tables for mixture problem. The first column is reserved for the ingredients and the next three columns will always have the headings of Amount, Price or Percent, and Total.

Amount – will identify either a weight or a capacity. The last cell for this column is found by adding the individual ingredients together. Often times the "mixture cell" will be given and the ingredients can be declared using the process discussed in the last section using the concept "The sum of two numbers is ..."

Price or percent – in many cases these values are given to the reader point blank. The last cell for this column will normally fall between the values for the individual ingredients.

Total – the last column can be found by multiplying the cells horizontally for both Amount and Price or Percent. Additionally, the equations are often found by adding the totals for each ingredient and setting the result equal to the last cell under this column heading. This is the reason for the hash marks on the last line in the table. It helps to identify the equation.

8. Walnuts that cost $4.05 per kilogram were mixed with cashews that cost $7.25 per kilogram. How many kilograms of each were used to make a 50-kilogram mixture costing $6.25 per kilogram? Round to the nearest tenth.

Ingredients	Amount kg	Price $	Total
walnuts	x	4.05	4.05x
cashews	50 – x	7.25	362.50 – 7.25x
mixture	50	6.25	312.50

Total for walnut + Total for cashews = Total for mixture

$4.05x + 362.50 - 7.25x = 312.50$

$-3.20x + 362.5 = 312.5$

$-3.20x = 312.5 - 362.5$

$-3.20x = -50$

$x = 15\frac{5}{8}$

walnuts = 15.6 kg
cashews = 34.4 kg

9. A tea mixture was made from 30 lb of tea costing $6.00 per pound and 70 lb of tea costing $3.20 per pound Find the cost per pound of the tea mixture.

Ingredients	Amount lbs	Price $	Total
Tea A	30	6.00	180
Tea B	70	3.20	224
Mixture	100	x	100x

Note: A and B are used as dummy variables, or variables used as place holders, they are not used in the equation. From this problem it should be understood that 100 cam from adding the weights for the individual ingredients and that X should fall between $3.20 and $6.00.

Total for Tea A + Total for Tea B = Total for Mixture
180 + 224 = 100x
404 = 100x
4.04 =x

Mixture should sell for $4.04 per pound.

10. A grocer combines 50 gal of cranberry juice that costs $3.50 per gallon with apple juice that costs $2.50 per gallon. How many gallons of apple juice must be used to make cranapple juice that costs $2.75 per gallon?

Ingredients	Amount gal	Price $	Total
cranberry	50	3.50	175
apple juice	x	2.50	2.50x
cranapple	x + 50	2.75	2.75x + 137.50

Total for cranberry + Total for apple juice = Total for Mixture

$175 + 2.50x = 2.75x + 137.50$

$175 - 137.50 = 2.75x - 2.50x$

$37.5 = .25x$

$150 = x$

150 gallons of apple juice should be added to the mixture.

11. A pharmacist has 150 dL of a 25% solution of peroxide in water. How many deciliters of pure peroxide should be added to obtain a 40% solution?

Ingredients	Amount dL	Percent %	Total
solution peroxide	150	25	3750
pure peroxide	x	100	100x
new solution peroxide	x + 150	40	40x +6000

Total for original + Total for pure = Total for new

$3750 + 100x = 40x + 6000$

$100x - 40x = 6000 - 3750$

$60x = 2250$

$x = 37\frac{1}{2}$

37.5 dL of pure peroxide should be added to the mixture.

<u>Coin and Stamp Problems</u>

<u>Notes regarding coin and stamp word problems</u>

These problems are actually a subset to the mixture problems. This means that the column headings are practically identical. Students should obviously know the value of coins. In this section equations can be found either in the first or the last algebraic columns. declaring variables can sometimes pose more of a challenge in this section. As a general rule one should start by declaring a variable for the item referred to most often.

12. A coin bank contains 25 coins in nickels, dimes, and quarters. There are twice as many dimes as quarters. The value of the coins is $2.15. How many **dimes** are in the bank?

Coins	Amount # coin	Value ¢	Total Value
Nickels	25 – 3q	5	125 – 15q
Dimes	2q	10	20q
Quarters	q	25	25q
	25		215

Note: The expression for nickels was created by taking the total number of coins and subtracting out the number of coins for both dimes and quarters.

Total value nickels + Total value dimes + Total value quarters = Total value of coins in bank

$125 - 15q + 20q + 25q = 215$

$125 + 30q = 215$

$30q = 215 - 125$

$30q = 90$

$q = 3$

There are 6 dimes in the bank.

13. A collection of stamps consists of 3¢ stamps, 5¢ stamps, and 7¢ stamps. There are six more 3¢ stamps than 5¢ stamps, and two more 7¢ stamps than 3¢ stamps. The total value of the stamps is $1.94. How many 3¢ stamps are in the collection?

Stamps	Amount # stamps	Value ¢	Total Value
Stamp A	x	3	3x
Stamp B	x – 6	5	5x –30
Stamp C	x + 2	7	7x + 14
			194

Total value A stamps + Total value B stamps + Total value C stamps = Total value

$3x + 5x - 30 + 7x + 14 = 194$

$15x - 16 = 194$

$15x = 194 + 16$

$15x = 210$

$x = 14$

There are 14 stamps @ 3¢

Simple Interest Problems

Notes regarding simple interest word problems

All of these problem are adapted to the chart method by using the formula I =prt. However, the order for the column headings will be p, r, t, and then I.

p – represents the principal amount to be invested. This is the start up money
r – represents the annual interest rate (it should be written as a decimal)
t – represents the number of years for the investment
I – represents the money earned from the investment
Avoid confusion with the types of investments by using dummy variables for the types of investments. Also recall that investment amounts are to be reported under the principal column and not the interest earned column.

14. Paul Young is investing \$6000 in two accounts, part at 4.5% and the remainder at 6%. If the total annual interest earned from the two accounts is \$279, how much did Paul deposit at each rate?

Accounts	p	r	t	I
Investment A	x	.045	1	.045x
Investment B	6000 – x	.06	1	360 –.06x
total	6000			279

Total Interest A + Total Interest B = Total Interest earned

$$.045x + 360 - .06x = 279$$
$$-.015x + 360 = 279$$
$$-.015x = 279 - 360$$
$$-.015x = -81$$
$$x = 5400$$

\$5400 invested @ 4.5%
\$600 invested @ 6%

15. An investment of $4500 is made at 7.8%. How much additional money must be invested at an 11% rate so that the total interest earned is equal to 9.5% of the total amount invested?

Accounts	p	r	t	I
Investment A	4500	.078	1	351
Investment B	x	.11	1	.11x
total	x + 4500	.095	1	.095x + 427.5

Note: Principal for investment A was given, principal for investment B was unknown, so the total is found by adding those principal amounts together.

Also remember that the last cell for rate usually falls between the two interest rates if it is needed. In other words, "P" is like "Amount", "r" is like "Percent or Price", "t" is almost always 1, and "I" is like "Total".

Total Interest A + Total Interest B = Total Interest earned

$351 + .11x = .095x + 427.5$

$.11x - .095x = 427.5 - 351$

$.015x = 76.5$

$x = 5100$

$5100 needs to be invested at 11%

Distance Rate Time problems

Notes regarding distance word problems

All of these problem are adapted to the chart method by using the formula D=rt. However, the order for the column headings will be r, t, and then D.

r – represents the rate of motion
t – represents the time the object is in motion
D– represents the distance traveled

There are a few concepts to remember with these problems. Most of the equations will be written using an intuitive approach regarding each scenario. These scenarios can be broken down into the following examples.

First: Objects traveling in opposite directs will span a given distance. It does not matter whether the objects are traveling toward one another or traveling away from a single starting point. As long as each object is traveling in opposite directions the equation should be $D_a + D_b = D_{total}$

Second: If one object gets a head start and the second object <u>*overtakes*</u> *the first then it should stand to reason that* $D_a = D_b$.

Third: If an object travels to one destination then turns around and uses the same path for a return trip it should stand to reason that $D_a = D_b$.

Fourth: If two objects are traveling in the same direction at the same time but at different speeds then as time elapses it will generate different distances. So the equation, what makes the two distances equal would be generated from, $D_{faster\_object} = D_{slower\_object} + Dist\_apart$

16. Two airplanes leave Dallas at the same time and fly in **opposite** directions. One airplane travels 80 miles per hour faster than the other. After three hours, they are 2940 miles apart. What is the rate of each airplane?

Objects	Rate mph	Time	Distance
Plane A	x + 80	3	3x + 240
Plane B	x	3	3x
Total			2940

Total Distance A + Total Distance B = Total Distance

$3x + 240 + 3x = 2940$

$6x + 240 = 2940$

$6x = 2940 - 240$

$6x = 2700$

$x = 450$

Plane A travels at 530 mph.
Plane B travels at 450 mph.

17. A commuter plane flew to a small town from a major airport at an average speed of 300 mph. The average speed on the return trip was 200 mph. What is the **distance** between the two airports if the total flying time was 4 hours?

Objects	Rate	Time	Distance
Going	300	t	300t
Returning	200	4 – t	800 – 200t
Total		4	

Distance Going = Distance Returning

$300t = 800 - 200t$

$300t + 200t = 800$

$500t = 800$

$t = 1\frac{3}{5} hours$

Distance = 300 (1.6)
Distance = 480 miles

18. A plane leaves an airport at 3 p.m. At 4 p.m. another plane leaves the same airport traveling in the same direction at a speed 150 mph faster than the first plane. Four hours after the first plane takes off, the second plane is 250 miles ahead of the first plane. How far did the second plane travel?

Objects	Rate mph	Time	Distance
Plane A	r	4	4r
Plane B	r + 150	3	3r + 450
Total			

Total Distance A = Total Distance B + Difference in Distance

$3r + 450 = 4r + 250$

$450 - 250 = 4r - 3r$

$200 = r$

PLane B distance = (200 + 150)*3
= (350)*3
= 1050 miles

Plane B traveled 1,050 miles.

16. At 1:30 p.m., an airplane leaves Tucson for Baltimore, a distance of 2240 miles. The plane files at 280 miles per hour. A second airplane leaves Tucson at 2:15 p.m., and is scheduled to land in Baltimore 15 minutes before the first airplane. At what rate must the second airplane travel to arrive on schedule?

Objects	Rate mph	Time	Distance
Plane A	280	t	280t
Plane B	r	t – 1	
Total			

Note: There was enough information given to figure out how long it takes plane a to travel the distance. Once that time is figured out, it can be used to determine the speed for the second plane.

Distance Plane A = 2240
280t = 2240
t = 8

Distance Plane B = 2240
7r = 2240
r = 320 mph
Plane B = 320 mph