# Simplifying Radicals

#1 $\sqrt{20}$

$\sqrt{2^2 \cdot 5}$

$2\sqrt{5}$

#2 $\sqrt{75}$

$\sqrt{5^2 \cdot 3}$

$5\sqrt{3}$

#3 $3\sqrt{32}$

$3\sqrt{4^2 \cdot 2}$

$12\sqrt{2}$

#4 $5\sqrt{27}$

$5\sqrt{3^2 \cdot 3}$

$15\sqrt{3}$

#5 $\sqrt{90A^2B^5}$

$\sqrt{3^2 \cdot 2 \cdot 5\, A^2 B^2 B^2 B}$

$3/A/B^2 \sqrt{10B}$

#6 $\sqrt{162x^3y^4z^6}$

$\sqrt{9^2 \cdot 2\, x^2 x\, y^2 y^2\, z^2 z^2 z^2}$

$9/x/y^2/z^3/\sqrt{2x}$

#7 $\frac{4\sqrt{3}}{\sqrt{2}}$

$\frac{4\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{4\sqrt{6}}{2} = 2\sqrt{6}$

#8 $\frac{2\sqrt{6}}{3\sqrt{2}}$

$\frac{2\sqrt{6}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{2\sqrt{2^2 \cdot 3}}{3 \cdot 2}$

$\frac{2\sqrt{3}}{3}$

#9 $\frac{3\sqrt{2}}{4-\sqrt{7}}$

$\frac{3\sqrt{2}}{4-\sqrt{7}} \cdot \frac{4+\sqrt{7}}{4+\sqrt{7}}$

$\frac{3\sqrt{2}(4+\sqrt{7})}{16-7}$

$\frac{\cancel{3}\sqrt{2}(4+\sqrt{7})}{\cancel{9}\ 3}$

$\frac{4\sqrt{2}+\sqrt{14}}{3}$

#10 $\frac{12}{\sqrt{3}-\sqrt{6}}$

$\frac{12}{\sqrt{3}-\sqrt{6}} \cdot \frac{\sqrt{3}+\sqrt{6}}{\sqrt{3}+\sqrt{6}}$

$\frac{12(\sqrt{3}+\sqrt{6})}{3-6}$

$\frac{4\ \cancel{12}(\sqrt{3}+\sqrt{6})}{-\cancel{3}\ 1}$

$-4\sqrt{3} - 4\sqrt{6}$

# Operations with Radicals

#1 $3\sqrt{5} + 2\sqrt{2} - 11\sqrt{5}$

$-8\sqrt{5} + 2\sqrt{2}$

#2 $3\sqrt{20} + 2\sqrt{8} - 2\sqrt{45} - 5\sqrt{18}$

$3\sqrt{2^2 \cdot 5} + 2\sqrt{2^2 \cdot 2} - 2\sqrt{3^2 \cdot 5} - 5\sqrt{3^2 \cdot 2}$

$6\sqrt{5} + 4\sqrt{2} - 6\sqrt{5} - 15\sqrt{2}$

$-11\sqrt{2}$

#3 $2\sqrt{3}(3\sqrt{2} - 4\sqrt{6})$

$6\sqrt{6} - 8\sqrt{3^2 \cdot 2}$

$6\sqrt{6} - 24\sqrt{2}$

#4 $\sqrt{5}(3\sqrt{15} + \sqrt{10})$

$3\sqrt{5^2 \cdot 3} + \sqrt{5^2 \cdot 2}$

$15\sqrt{3} + 5\sqrt{2}$

#5 $(2\sqrt{3}-5\sqrt{10})(2\sqrt{3}+3\sqrt{15})$

$4\cdot 3 + 6\sqrt{3^2\cdot 5} - 10\sqrt{30} - 15\sqrt{5^2\cdot 6}$

$12 + 18\sqrt{5} - 10\sqrt{30} - 75\sqrt{6}$

#6 $(\sqrt{6}-2\sqrt{8})(\sqrt{24}+3\sqrt{2})$

$\sqrt{6^2\cdot 2^2} + 3\cdot\sqrt{2^2\cdot 3} - 2\sqrt{8^2\cdot 3} - 6\sqrt{2^2\cdot 2^2}$

$12 + 6\sqrt{3} - 16\sqrt{3} - 24$

$-10\sqrt{3} - 12$

Solving Radical Equations

#1 $\sqrt{12-k} = k$

$\left[\sqrt{12-k}\right]^2 = k^2$

$12-k = k^2$

$0 = k^2+k-12$

$0 = (k-3)(k+4)$

$k=3 \qquad k=-4$

EXTRANEOUS

$k=3$

#2 $p = \sqrt{-1-2p}$

$[p]^2 = \left[\sqrt{-1-2p}\right]^2$

$p^2 = -1-2p$

$p^2+2p+1=0$

$(p+1)(p+1)=0$

$\underbrace{p=-1 \quad p=-1}_{\text{EXTRANEOUS}}$

$p = \{\}$

#3 $\sqrt{-16+10R} = R$

$\left[\sqrt{-16+10R}\right]^2 = [R]^2$

$-16+10R = R^2$

$0 = R^2-10R+16$

$0 = (R-8)(R-2)$

$R=8 \qquad R=2$

$R = \{2, 8\}$

#4 $-2+\sqrt{6g+19} = g$

$\left[\sqrt{6g+19}\right]^2 = [g+2]^2$

$6g+19 = g^2+4g+4$

$0 = g^2-2g-15$

$0 = (g-5)(g+3)$

$g=5 \qquad g=-3$

EXTRANEOUS

$g = \{5\}$

#5 $x-7=\sqrt{3x-21}$

$(x-7)^2=\left[\sqrt{3x-21}\right]^2$

$x^2-14x+49=3x-21$

$x^2-17x+70=0$

$(x-7)(x-10)=0$

$x=7 \quad x=10 \; =0$

$x=\{7,10\}$

#6 $m-4=\sqrt{10-3m}$

$(m-4)^2=\left(\sqrt{10-3m}\right)^2$

$m^2-8m+16=10-3m$

$m^2-5m+6=0$

$(m-2)(m-3)=0$

$\underbrace{m=2 \quad m=3}_{\text{EXTRANEOUS}}$

$m=\{\,\}$

QUADRATIC FORMULA $X=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

#1 $A^2-5A-9=0$

A=1
B=-5
C=-9

$X=\dfrac{5\pm\sqrt{25-4(1)(-9)}}{2(1)}$

$=\dfrac{5\pm\sqrt{25+36}}{2}$

$=\dfrac{5\pm\sqrt{61}}{2}$

$(5-\sqrt{(61)})/2 \quad , \quad (5+\sqrt{(61)})/2$

$-1.4 \quad , \quad 6.4$

$A=\{-1.4, 6.4\}$ IRRATIONAL

#2 $8y^2+10y+3=0$

A=8
B=10
C=3

$y=\dfrac{-10\pm\sqrt{100-4(8)(3)}}{2(8)}$

$=\dfrac{-10\pm\sqrt{100-96}}{16}$

$=\dfrac{-10\pm\sqrt{4}}{16}$

$=\dfrac{-10\pm 2}{16}$

$\dfrac{-10-2}{16} \quad , \quad \dfrac{-10+2}{16}$

$-12/16 \quad , \quad -8/16$

$y=\{-3/4, -1/2\}$ RATIONAL

QUADRATIC FORMULA $\quad x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

#3 $\quad 2R^2 + 5R - 1 = 0$

$A=2$
$B=5$
$C=-1$

$$R = \frac{-5 \pm \sqrt{25-4(2)(-1)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25+8}}{4}$$

$$= \frac{-5 \pm \sqrt{33}}{4}$$

$\left(-5-\sqrt{(33)}\right)/4 \quad , \quad \left(-5+\sqrt{(33)}\right)/4$

$-2.7 \quad , \quad .2$

$R = \{-2.7, .2\}$ IRRATIONAL

#4 $\quad 3w^2 + 8w + 2 = 0$

$A=3$
$B=8$
$C=2$

$$w = \frac{-8 \pm \sqrt{64-4(3)(2)}}{2(3)}$$

$$= \frac{-8 \pm \sqrt{64-24}}{6}$$

$$= \frac{-8 \pm \sqrt{40}}{6}$$

$\left(-8-\sqrt{40}\right)/6 \quad , \quad \left(-8+\sqrt{(40)}\right)/6$

$-2.4 \quad , \quad -.3$

$w = \{-2.4, -.3\}$ IRRATIONAL

#5 $\quad 2B^2 - B - 15 = 0$

$A=2$
$B=-1$
$C=-15$

$$= \frac{1 \pm \sqrt{1-4(2)(-15)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{1+120}}{4}$$

$$= \frac{1 \pm \sqrt{121}}{4}$$

$$= \frac{1 \pm 11}{4}$$

$\dfrac{1-11}{4} \quad , \quad \dfrac{1+11}{4}$

$-2\frac{1}{2} \quad , \quad 3$

$B = \{-2\frac{1}{2}, 3\}$ RATIONAL

$2x^2 + 9x + 13 = 0$

$A=2$
$B=9$
$C=13$

$$x = \frac{-9 \pm \sqrt{81-4(2)(13)}}{2(2)}$$

$$= \frac{-9 \pm \sqrt{81-104}}{4}$$

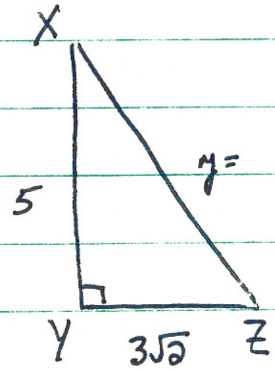
$$= \frac{-9 \pm \sqrt{-23}}{4}$$

CAN NOT HAVE NEGATIVE UNDER RADICAL

$\{\}$ DOES NOT EXIST

PYTHAGOREAN THEOREM

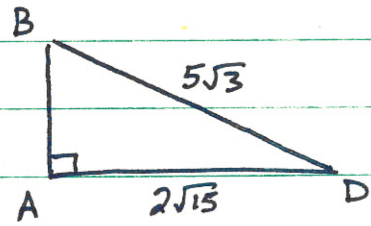
#1




$$l^2 + l^2 = h^2$$
$$5^2 + (3\sqrt{2})^2 = y^2$$
$$25 + 9 \cdot 2 = y^2$$
$$25 + 18 = y^2$$
$$43 = y^2$$

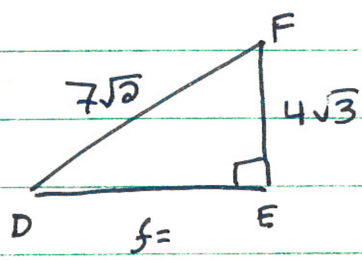
$$\sqrt{43} = y$$

#2




$$l^2 + l^2 = h^2$$
$$(2\sqrt{15})^2 + d^2 = (5\sqrt{3})^2$$
$$4 \cdot 15 + d^2 = 25 \cdot 3$$
$$60 + d^2 = 75$$
$$d^2 = 75 - 60$$
$$d^2 = 15$$
$$d = \sqrt{15}$$

#3




$$l^2 + l^2 = h^2$$
$$(4\sqrt{3})^2 + f^2 = (7\sqrt{2})^2$$
$$16 \cdot 3 + f^2 = 49 \cdot 2$$
$$48 + f^2 = 98$$
$$f^2 = 98 - 48$$
$$f^2 = 50$$
$$f = \sqrt{50}$$
$$f = \sqrt{5^2 \cdot 2}$$
$$f = 5\sqrt{2}$$

* ALWAYS SIMPLIFY RADICALS IF POSSIBLE

* TRIANGLE EXISTENCE POSTULATE

THE SUM AND DIFFERENCE FOR THE LEGS OF A TRIANGLE WILL CREATE A RANGE OF VALUES THAT THE HYPOTENEUSE MUST FALL WITHIN IN ORDER TO CREATE A TRIANGLE.

#1 5, 7, 9

$7-5=2$

$7+5=12$

SINCE 9 FALLS WITHIN INTERVAL FROM 2 - 12 THE TRIANGLE EXISTS

$$l^2+l^2=h^2$$
$$5^2+7^2 \overset{?}{=} 9^2$$
$$25+49 \overset{?}{=} 81$$
$$74 < 81$$

$\therefore$ OBTUSE TRIANGLE

#2 14, 48, 50

$48-14=34$

$48+14=62$

SINCE 50 FALLS WITHIN INTERVAL FROM 34 - 62 THE TRIANGLE EXISTS

$$l^2+l^2=h^2$$
$$14^2+48^2 \overset{?}{=} 50^2$$
$$196+2304 \overset{?}{=} 2500$$
$$2500 = 2500$$

$\therefore$ RIGHT TRIANGLE

#3 $3\sqrt{5}$, $5\sqrt{6}$, 15

$5\sqrt{6}-3\sqrt{5} \approx 5.5$

$5\sqrt{6}+3\sqrt{5} \approx 18.96$

SINCE 15 FALLS WITHIN INTERVAL 5.5 - 19.0 THE TRIANGLE EXISTS

$$l^2+l^2=h^2$$
$$(3\sqrt{5})^2+(5\sqrt{6})^2 \overset{?}{=} 15^2$$
$$9.5+25.6 \overset{?}{=} 225$$
$$45+150 \overset{?}{=} 225$$
$$195 < 225$$

$\therefore$ OBTUSE TRIANGLE

#4 $2\sqrt{3}, 3\sqrt{5}, \sqrt{57}$

$3\sqrt{5} - 2\sqrt{3} \approx 3.2$

$3\sqrt{5} + 2\sqrt{3} \approx 10.2$

SINCE $\sqrt{57} \approx 7.5$ FALLS WITHIN

INTERVAL $3.2 - 10.2$

THE TRIANGLE EXISTS

$l^2 + l^2 = h^2$

$(2\sqrt{3})^2 + (3\sqrt{5})^2 \stackrel{?}{=} (\sqrt{57})^2$

$4 \cdot 3 + 9 \cdot 5 \stackrel{?}{=} (\sqrt{57})^2$

$12 + 45 \stackrel{?}{=} 57$

$57 = 57$

$\therefore$ TRIANGLE IS RIGHT

#5 $5, 7, 14$

$7 - 5 = 2$

$7 + 5 = 12$

14 DOES NOT FALL

WITHIN INTERVAL $2 - 12$

SO TRIANGLE DOES NOT EXIST

* NOTE IF $l^2 + l^2 > h^2$ THE TRIANGLE IS ACUTE. THERE WERE NO EXAMPLES OF THIS ON THE WORKSHEET.

<u>DISTANCE FORMULA</u> $d = \sqrt{\Delta x^2 + \Delta y^2}$

#1 $6 \left\langle \begin{matrix} (-2, 4) \\ (4, -2) \end{matrix} \right\rangle -6$

$d = \sqrt{6^2 + (-6)^2}$

$= \sqrt{36 + 36}$

$= \sqrt{72}$

$= \sqrt{6^2 \cdot 2}$

$= 6\sqrt{2}$

#2 $-5 \left\langle \begin{matrix} (3, 1) \\ (-2, -2) \end{matrix} \right\rangle -3$

$d = \sqrt{-5^2 + -3^2}$

$= \sqrt{25 + 9}$

$= \sqrt{34}$

#3 $\overset{+2}{\phantom{x}}\left\langle \begin{matrix}(3, 6)\\(5, -5)\end{matrix}\right\rangle -11$

$$d = \sqrt{2^2 + (-11)^2}$$
$$= \sqrt{4 + 121}$$
$$= \sqrt{125}$$
$$= 5\sqrt{5}$$

#4 $\overset{+9}{\phantom{x}}\left\langle \begin{matrix}(-2, 4)\\(7, -8)\end{matrix}\right\rangle -12$

$$d = \sqrt{9^2 + (-12)^2}$$
$$= \sqrt{81 + 144}$$
$$= \sqrt{225}$$
$$= 15$$

NOT ON WORKSHEET

#5 $\overset{+4\sqrt{2}}{\phantom{x}}\left\langle \begin{matrix}(3\sqrt{2}, 4)\\(7\sqrt{2}, 9)\end{matrix}\right\rangle +5$

$$d = \sqrt{(4\sqrt{2})^2 + 5^2}$$
$$= \sqrt{16 \cdot 2 + 25}$$
$$= \sqrt{32 + 25}$$
$$= \sqrt{57}$$

## Special Right Triangles

30–60–90

SL LL HYP

$1 : 1\sqrt{3} : 2$

45–45–90

L : L : HYP

$1 : 1 : 1\sqrt{2}$

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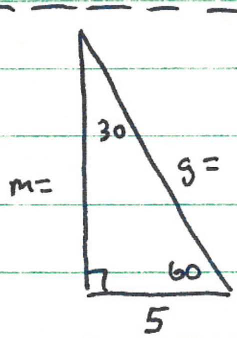
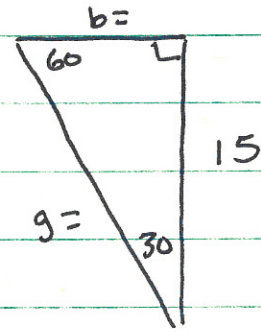
#1




$5 : 5\sqrt{3} : 10$

$m = 5\sqrt{3}$, $g = 10$

#2




$1 : 1\sqrt{3} : 2$

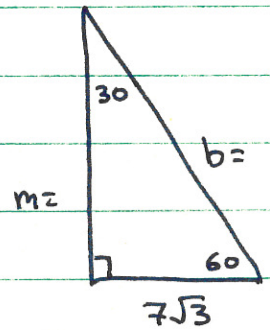
15

$\frac{15}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{3}$

$5\sqrt{3}$

$b = 5\sqrt{3}$, $g = 10\sqrt{3}$

#3




$1 : 1\sqrt{3} : 2$

$7\sqrt{3} : 21 : 14\sqrt{3}$

$(7\sqrt{3})(1\sqrt{3})$

$7 \cdot 3$

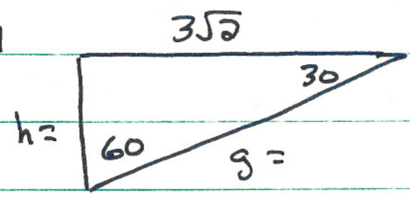
$21$

$2(7\sqrt{3})$

$14\sqrt{3}$

$m = 21$ $b = 14\sqrt{3}$

#4




$1 : 1\sqrt{3} : 2$

$\sqrt{6} : 3\sqrt{2} : 2\sqrt{6}$

$2(\sqrt{6})$

$2\sqrt{6}$

$\frac{3\sqrt{2}}{\sqrt{3}}$

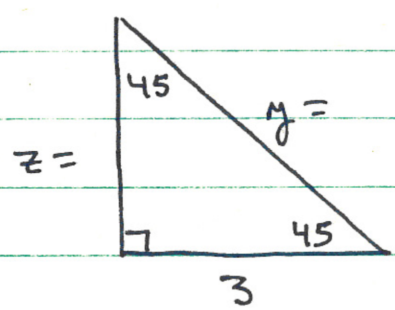
$\frac{3\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{3\sqrt{6}}{3}$

$\sqrt{6}$

$h = \sqrt{6}$ $g = 2\sqrt{6}$

#5

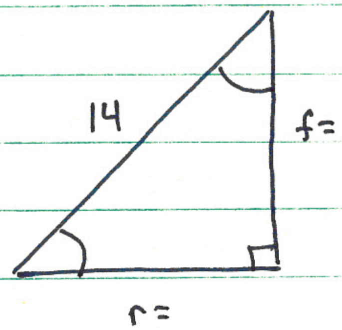



$1 : 1 : 1\sqrt{2}$

$3 : 3 : 3\sqrt{2}$

$z = 3$ $y = 3\sqrt{2}$

#6




$1 : 1 : 1\sqrt{2}$

$7\sqrt{2} : 7\sqrt{2} : 14$

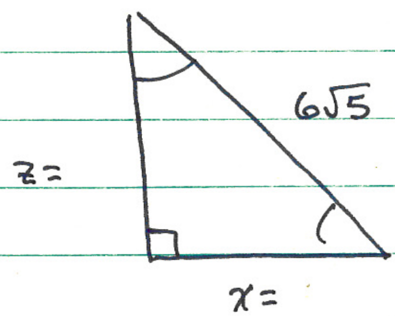
$\downarrow$

$\frac{14}{\sqrt{2}}$

$\frac{14}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{14\sqrt{2}}{2} = 7\sqrt{2}$

$r = 7\sqrt{2}$ $f = 7\sqrt{2}$

#7




$1 : 1 : 1\sqrt{2}$

$3\sqrt{10} : 3\sqrt{10} : 6\sqrt{5}$

$\frac{6\sqrt{5}}{\sqrt{2}}$

$\frac{6\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{10}}{2} = 3\sqrt{10}$

$z = 3\sqrt{10}$ $x = 3\sqrt{10}$