A. Slope - Intercept Form of Linear Equations

I. Characteristics

a. Slope = $m = \frac{\Delta y}{\Delta x}$

i. When slope is positive, the line will "go up" or incline
When slope is negative, the line will "go down" or decline

ii. If the slope is an improper fraction, the line will rise or fall in a steep fashion.
If the slope is a proper fraction, the line will rise or fall in a more shallow fashion.

b. y-intercept

i. When considering the constant in the equation, it will indicate where the line intersects the y-axis. This is most commonly written as an ordered pair.

II. Form of Equation

a. $y = mx + b$, where m = slope and b = y-coordinate for y-intercept

b. $y = \frac{\Delta y}{\Delta x}x + c$, where $\frac{\Delta y}{\Delta x}$ = slope and c = y-coordinate for y-intercept

III. Examples

Ex 1. $y = \frac{2}{7}x + 5 \rightarrow$ y-int = $(0, 5)$

↳ Slope = $\frac{2}{7} \Rightarrow$ shallow / incline

Ex 2. $y = -\frac{8}{3}x + 2 \rightarrow$ y-int $(0, 2)$

↳ Slope = $-\frac{8}{3} \Rightarrow$ steep / decline

Ex 3. $y = -\frac{1}{2}x - 7 \rightarrow$ y-int $(0, -7)$

↳ Slope = $-\frac{1}{2} \Rightarrow$ shallow / decline

IV. WRITING SLOPE INTERCEPT FORM EQUATIONS FROM TWO POINTS.

Ex 1 $(-3, 7)$ $(0, 5)$

[1] FIND SLOPE $+3 < \begin{matrix}(-3,7)\\(0,5)\end{matrix} > -2$ $\quad \frac{\Delta y}{\Delta x} = \frac{-2}{3}$

[2] USE FORM $\quad y = \frac{\Delta y}{\Delta x} x + c$

$y = \frac{-2}{3} x + c$: SUBSTITUTE $-2/3$ IN FOR SLOPE

$7 = \frac{-2}{3}(-3) + c$: PLUG IN EITHER SET OF (x, y) VALUES

$7 = 2 + c$: SOLVE FOR C

$5 = c$

$\boxed{y = -2/3 x + 5}$: WRITE THE <u>SLOPE</u> AND <u>INTERCEPT</u> FORM OF EQUATION

Ex 2 $(4, 2)$ $(8, 5)$

[1] FIND SLOPE $+4 < \begin{matrix}(4,2)\\(8,5)\end{matrix} > +3$ $\quad \frac{\Delta y}{\Delta x} = \frac{3}{4}$

[2] USE FORM

$y = \frac{\Delta y}{\Delta x} x + c$: SUBSTITUTE $3/4$ IN FOR SLOPE

$y = 3/4 x + c$: PLUG IN A POINT FOR x & y THEN SOLVE FOR C,

$2 = 3/4 (4) + c$

$2 = 3 + c$

$-1 = c$: WRITE EQUATION IN SLOPE-INT FORM

$\boxed{y = 3/4 x - 1}$

Ex 3 $(4, 10)$ $(2, 7)$

1 FIND SLOPE $-2 < \begin{matrix}(4,10)\\(2,7)\end{matrix} > -3$ $\frac{\Delta y}{\Delta x} = \frac{-3}{2}$

2 USE FORM $y = \frac{\Delta y}{\Delta x} x + c$

$y = \frac{-3}{2} x + c$: SUBSTITUTE SLOPE

$7 = \frac{-3}{2}(2) + c$: PLUG IN POINT

$7 = -3 + c$

$10 = c$: Solve for c

$\boxed{y = -\frac{3}{2} x + 10}$: WRITE SLOPE - INTERCEPT form of EQUATION

V. WRITING SLOPE INTERCEPT FORM WHEN GIVEN A SLOPE AND A POINT.

Ex 1 $m = 3/4$ $(12, -3)$

$y = \frac{\Delta y}{\Delta x} x + c$: START BY PLUGGING IN VALUE of SLOPE

$y = \frac{3}{4} x + c$: THEN PUT IN x & y VALUES FROM POINT

$-3 = \frac{3}{4}(12) + c$: Solve for c

$-3 = 9 + c$

$-12 = c$

$\boxed{y = 3/4 x - 12}$: WRITE EQ IN SLOPE - INT form

Ex 2 $m = -2/5$

$(3, 5)$

$y = \frac{\Delta y}{\Delta x} x + c$

$y = -\frac{2}{5} x + c$: SUBSTITUTE SLOPE

$5 = -\frac{2}{5}(3) + c$: PLUG IN x & y VALUES

$5 = \frac{-6}{5} + c$

$\frac{25}{5} + \frac{6}{5} = c$

$\frac{31}{5} = 6\frac{1}{5} = c$: SOLVE FOR C [C IS WHOLE OR MIXED NUMBER]

* SLOPE IS ALWAYS A FRACTION

$y = -2/5x + 6\frac{1}{5}$

B. Standard Form of Linear Equation

I. Characteristics

a. Slope = $\frac{-A}{B}$ using this form

b. Intercepts

i. y-intercept found by substituting zero in for "x" [Looking for y-value]

ii. x-intercept found by substituting zero in for "y" [Looking for x-value]

II. Form of Equation

a. $Ax + By = C$

i. x term written first, y-term written second, then equals constant value.

ii. A, B, and C must be integers [no fractions]

iii. A must be positive

III. Writing Standard Form Equations from Slope Intercept.

Ex 1 $y = \frac{2}{3}x - 4$: Given equation

$-\frac{2}{3}x + y = -4$: Move x term into first position.

$-3\left[-\frac{2}{3}x + y = -4\right]$: Multiply by appropriate value to remove fraction(s) and make leading term positive.

$\boxed{2x - 3y = 12}$ Equation written in standard form

Ex 2 $y = -\frac{3}{5}x + 2$: GIVEN EQUATION

$\frac{3}{5}x + y = 2$: MOVE x TERM INTO FIRST POSITION

$5\left[\frac{3}{5}x + y = 2\right]$: MULTIPLY BY APPROPRIATE VALUE TO REMOVE FRACTION(S) AND KEEP LEADING TERM POSITIVE

$\boxed{3x + 5y = 10}$ EQUATION WRITTEN IN STANDARD FORM

Ex 3 $y = \frac{5}{2}x - 3\frac{1}{4}$: GIVEN EQUATION

$-\frac{5}{2}x + y = -\frac{13}{4}$: MOVE x TERM INTO FIRST POSITION
* ALSO TURN CONSTANT INTO IMPROPER FRACTION WHEN GIVEN AS MIXED NUMBER.

$-4\left[-\frac{5}{2}x + y = -\frac{13}{4}\right]$: MULTIPLY BY APPROPRIATE VALUE TO REMOVE FRACTIONS AND MAKE LEADING TERM POSITIVE. (COMMON DENOMINATOR)

$\boxed{10x - 4y = 13}$ EQUATION WRITTEN IN STANDARD FORM

V. Writing Slope Intercept form equations from Standard.

Ex 1 $5x - 3y = 9$: equation given

$-3y = -5x + 9$: Solve for y

$\frac{-3y}{-3} = \frac{-5x}{-3} + \frac{9}{-3}$: Divide each term by -3

$y = \frac{5}{3}x - 3$: Slope always a fraction / constant is mixed number or whole.

Ex 2 $3x + 2y = 10$: equation given

$2y = -3x + 10$: Solve for y

$\frac{2y}{2} = \frac{-3x}{2} + \frac{10}{2}$: Divide each term by 2

$y = -\frac{3}{2}x + 5$: Slope always a fraction

Ex 3 $7x - 4y = 11$: equation given

$-4y = -7x + 11$: Solve for y

$\frac{-4y}{-4} = \frac{-7x}{-4} + \frac{11}{-4}$: Divide each term by -4

$y = \frac{7}{4}x - 2\frac{3}{4}$: Slope always a fraction / constant mixed or whole number.

VI. STANDARD EQUATION FROM TWO POINTS

a. COULD WRITE EQUATION IN SLOPE INTERCEPT FORM FIRST AND THEN CONVERT TO STANDARD (PROCESS ALREADY DESCRIBED IN NOTES)

b. USE KNOWLEDGE OF STANDARD FORM.

Ex 1 $(3,7)$ $(-2,3)$

[1] FIND SLOPE $-5 < \begin{matrix}(3,7)\\(-2,3)\end{matrix} > -4$ $\quad \frac{\Delta y}{\Delta x} = \frac{4}{5}$

[2] USE FORM $Ax + By = C$

$4x - 5y = C$

PLUG IN x & y VALUES → $4(3) - 5(7) = C$

$12 - 35 = C$

$-23 = C$

$\boxed{4x - 5y = -23}$: WRITE STANDARD FORM.

* WHEN SLOPE IS POSITIVE STANDARD FORM HAS (−) IN FRONT OF y-TERM

* WHEN SLOPE IS NEGATIVE STANDARD FORM HAS (+) IN FRONT OF y-TERM

Ex 2 $(-2,3)$ $(4,2)$

[1] FIND SLOPE $+6 < \begin{matrix}(-2,3)\\(4,2)\end{matrix} > -1$ $\quad \frac{\Delta y}{\Delta x} = -\frac{1}{6}$

[2] USE FORM $Ax + By = C$

$1x + 6y = C$: (−) SLOPE ⇒ (+) y TERM

$1(4) + 6(2) = C$: PLUG IN x & y VALUES

$4 + 12 = C$

$16 = C$: SOLVE FOR C

$\boxed{x + 6y = 16}$: WRITE STANDARD FORM EQ.

VII. Standard form from a point and a slope

a. Could write equation in slope-intercept form & then convert using process already described

b. Use knowledge of standard form

Ex 1 $m = \frac{3}{5}$ $(2, 7)$

Use form $Ax + By = C$

$3x - 5y = C$: (+) slope $\Rightarrow$ (−) y-term

$3(2) - 5(7) = C$: plug in point

$6 - 35 = C$

$-29 = C$: solve for C

$\boxed{3x - 5y = -29}$: write equation in standard form

Ex 2 $m = \frac{-5}{2}$ $(3, 1)$

Use form $Ax + By = C$

$5x + 2y = C$: (−) slope $\Rightarrow$ (+) y-term

$5(3) + 2(1) = C$: plug in point

$15 + 2 = C$

$17 = C$: solve for C

$\boxed{5x + 2y = 17}$: write standard form of eq.

C. Point-Slope Form

I. Forms of Equation

a. $(y-h) = m(x-k)$

b. $(y-h) = \frac{\Delta y}{\Delta x}(x-k)$

Slope $= m = \frac{\Delta y}{\Delta x}$

point (h, k)

II. Writing Equation

Ex 1 $m = 3/4$, $(2, 7)$ $\Rightarrow$ $(y-7) = 3/4(x-2)$

Ex 2 $m = -4/5$, $(5, -3)$ $\Rightarrow$ $(y+3) = -4/5(x-5)$

Ex 3 $m = -2/3$, $(-2, 4)$ $\Rightarrow$ $y-4 = -2/3(x+2)$