A. Slope Intercept Form of Linear Equations $y = mx + b$ or $y = \frac{\Delta y}{\Delta x}x + c$

I. Characteristics

a. **Slope = m =** $\frac{\Delta y}{\Delta x}$ . . . (pitch of a line)

i. When the slope is ***positive***, the line will "go up" or **incline** as it is read from left to right.
When the slope is ***negative***, the line will "go down" or **decline** as it is read from left to right.

ii. If the slope is an **improper fraction**, the line will rise or fall in a **steep** fashion.
If the slope is a **proper fraction**, the line will rise or fall in a **shallow** fashion.

b. **y-intercept** (0, b) or (0, c) **. . .** (where the line crosses the y axis)

i. When considering the constant in this form, it will indicate where the line intersects the y axis.
This is most commonly written as an **ordered pair**.

II. Form of Equation

a. $y = mx + b$, where m = slope and b = y – coordinate for y-intercept.

b. $y = \frac{\Delta y}{\Delta x}x + c$, where $\frac{\Delta y}{\Delta x}$ = slope and c = y – coordinate for y-intercept.

III. Examples

Ex. 1 $y = \frac{2}{7}x + 5$ $\begin{cases} slope = \frac{2}{7}, \Rightarrow shallow/incline \\ y - \text{int} = (0,5) \end{cases}$

Ex. 2 $y = -\frac{8}{3}x + 2$ $\begin{cases} slope = -\frac{8}{3}, \Rightarrow steep/decline \\ y - \text{int} = (0,2) \end{cases}$

Ex. 3 $y = -\frac{1}{2}x - 7$ $\begin{cases} slope = -\frac{1}{2}, \Rightarrow shallow/decline \\ y - \text{int} = (0,-7) \end{cases}$

IV. <u>Writing Slope Intercept Form From Two Points</u>

Ex. 1 Given (–3, 7) and (0 , 5),

1) Find the **slope** $+3\left\langle\begin{matrix}(-3,7)\\(0,5)\end{matrix}\right\rangle-2$ so $\frac{\Delta y}{\Delta x}=-\frac{2}{3}$

2) Use Form $y=\frac{\Delta y}{\Delta x}x+c$

$y=-\frac{2}{3}x+c$: Substitute $-\frac{2}{3}$ in for the slope

$7=-\frac{2}{3}(-3)+c$: Plug in either set of (x , y) values

$7=2+c$: Solve for c

$5=c$

$y=-\frac{2}{3}x+5$:Write the **slope** and **intercept** form of equation.

Ex. 2 Given (4, 2) and (8 , 5),

1) Find the **slope** $+4\left\langle\begin{matrix}(4,2)\\(8,5)\end{matrix}\right\rangle+3$ so $\frac{\Delta y}{\Delta x}=\frac{3}{4}$

2) Use Form $y=\frac{\Delta y}{\Delta x}x+c$

$y=\frac{3}{4}x+c$: Substitute $\frac{3}{4}$ in for the slope

$2=\frac{3}{4}(4)+c$: Plug in either set of (x , y) values

$2=3+c$: Solve for c

$-1=c$

$y=\frac{3}{4}x-1$:Write the **slope** and **intercept** form of equation.

- *Slopes* should always be written as **proper or improper** fractions
- While *intercepts* are **proper fractions, mixed, or whole numbers** but never as improper fractions.

Ex. 3 Given (5, –3) and (3 , –8),

1) Find the **slope** $+3\left\langle\begin{matrix}(5,-3)\\(2,-8)\end{matrix}\right\rangle-5$ so $\frac{\Delta y}{\Delta x}=-\frac{5}{3}$

2) Use Form $y=\frac{\Delta y}{\Delta x}x+c$

$y=-\frac{5}{3}x+c$: Substitute $-\frac{5}{3}$ in for the slope

$-3=-\frac{5}{3}(5)+c$: Plug in either set of (x , y) values

$-3=-8\frac{1}{3}+c$: Solve for c

$-3+8\frac{1}{3}=c$

$y=-\frac{5}{3}x+5\frac{1}{3}$:Write the **slope** and **intercept** form of equation.

- *Slopes* should always be written as **proper or improper** fractions
- While *intercepts* are **proper fractions, mixed, or whole numbers** but never as improper fractions.

V. Writing Slope Intercept Equations When Given a Slope and a Point

Ex. 1 Given $m = \frac{3}{4}$ and (8 , –3),

1) Use Form $y = \frac{\Delta y}{\Delta x}x + c$

$y = \frac{3}{4}x + c$: Substitute $\frac{3}{4}$ in for the slope

$-3 = \frac{3}{4}(8) + c$: Plug in the only set of (x , y) values

$-3 = 6 + c$: Solve for c

$-3 - 6 = c$

$y = \frac{3}{4}x - 9$:Write the **slope** and **intercept** form of equation.

Ex. 2 Given $m = -\frac{2}{5}$ and (3 , 5),

1) Use Form $y = \frac{\Delta y}{\Delta x}x + c$

$y = -\frac{2}{5}x + c$: Substitute $-\frac{2}{5}$ in for the slope

$5 = -\frac{2}{5}(3) + c$: Plug in the only set of (x , y) values

$5 = -1\frac{1}{5} + c$: Solve for c

$5 + 1\frac{1}{5} = c$

$y = -\frac{2}{5}x + 6\frac{1}{5}$:Write the **slope** and **intercept** form of equation.

- *Slopes* should always be written as **proper or improper** fractions
- While *intercepts* are **proper fractions, mixed, or whole numbers** but never as improper fractions.

B. Standard Form of Linear Equations $Ax + By = C$

I. Characteristics

a. **Slope = m =** $\frac{-A}{B}$ . . . Using this form (Note: B and C are not equated to b, c from the previous form.

i. When the slope is ***positive***, the line will "go up" or **incline** as it is read from left to right.
When the slope is ***negative***, the line will "go down" or **decline** as it is read from left to right.

ii. If the slope is an **improper fraction**, the line will rise or fall in a **steep** fashion.
If the slope is a **proper fraction**, the line will rise or fall in a **shallow** fashion.

b. Intercepts

i. **y-intercept** (0, y) **. . .** (where the line crosses the y axis)
y-intercept is found by substituting zero in for "x" **[looking for y value].**
This is most commonly written as an **ordered pair**.

ii**. x-intercept** (x, 0) **. . .** (where the line crosses the x axis)
x-intercept is found by substituting zero in for "y" **[looking for x value].**
This is most commonly written as an **ordered pair**.

II. Form of Equation $Ax + By = C$

a. **X term** must be written first, **Y term** written second, then equals a **constant**
b. A, B, and C must be written as integers [NO FRACTIONS]
c. A must be positive.

III. Writing **Standard Form Equations** From **Slope Intercept Form Equations**.

Ex. 1 $y = \frac{2}{3}x - 4$: given Slope Intercept form of Line

$-\frac{2}{3}x + y = -4$: move x term into first position (changing sign of course)

$-3 \bullet \left[-\frac{2}{3}x + y = -4\right]$: multiply by – 3 to make leading term positive and remove fractions.

$2x - 3y = 12$: Standard Form Equation

Ex. 2 $y = -\frac{3}{5}x + 2$: given Slope Intercept form of Line

$\frac{3}{5}x + y = +2$: move x term into first position (changing sign of course)

$5 \bullet \left[\frac{3}{5}x + y = +2\right]$: multiply by 5 to make leading term positive and remove fractions.

$3x + 5y = 10$: Standard Form Equation

Ex. 3 $y = \frac{5}{2}x - 3\frac{1}{4}$: given Slope Intercept form of Line

$-\frac{5}{2}x + y = -\frac{13}{4}$: move x term into first position (changing sign of course)

Also change y-intercept into improper fraction when presented as mixed number.

$-4 \bullet \left[-\frac{5}{2}x + y = -\frac{13}{4}\right]$: multiply by –4 (common denominator) to make leading term positive and remove fractions.

$10x - 4y = 13$: Standard Form Equation

IV Writing **Slope Intercept Form** Equations from **Standard**

Ex. 1 $5x - 3y = 9$: given Standard Form of Line

$-3y = -5x + 9$: move x term across equal sign (changing sign of course)

$\frac{-3y}{-3} = \frac{-5x}{-3} + \frac{9}{-3}$: Divide by –3 to solve for y (or get "y" by itself)

$y = \frac{5}{3}x - 3$: Slope Intercept Equation

Ex. 2

$3x + 2y = 10$: given Standard Form of Line

$2y = -3x + 10$: move x term across equal sign (changing sign of course)

$\frac{2y}{2} = \frac{-3x}{2} + \frac{10}{2}$: Divide by 2 to solve for y (or get "y" by itself)

$y = -\frac{3}{2}x + 5$: Slope Intercept Equation

Ex. 3

$7x - 4y = 11$: given Standard Form of Line

$-4y = -7x + 11$: move x term across equal sign (changing sign of course)

$\frac{-4y}{-4} = \frac{-7x}{-4} + \frac{11}{-4}$: Divide by –4 to solve for y (or get "y" by itself)

$y = \frac{7}{4}x - 2\frac{3}{4}$: Slope Intercept Equation

Slope is always a fraction / intercept never improper

V Standard Equation From Two Points

A. Could write the equation in slope intercept form first then convert to standard
(Process already described in these notes)

B. Use knowledge of Standard Form to directly write equation (optional method)

Ex. 1 Given (3, 7) and (–2 , 3),

1) Find the **slope** $-5\left\langle \begin{matrix}(3\ ,\ 7)\\(-2,\ 3)\end{matrix} \right\rangle - 4$ so $\frac{\Delta y}{\Delta x} = \frac{4}{5}$

2) Use Form $Ax + By = C$:Think of it as Δy(x) + Δx(y) = C

$4x - 5y = C$: When the slope is **positive**, standard form has **(–)** in front of the y - term

$4(3) - 5(7) = C$: Plug in either set of (x , y) values

$12 - 35 = C$: Solve for C

$-23 = C$

$4x - 5y = -23$:Write the **Standard** Form of equation.

Ex. 2 Given (–2 , 3) and (4, 2)

1) Find the **slope** $+6\left\langle \begin{matrix}(-2,\ 3)\\(\ 4,\ 2)\end{matrix} \right\rangle - 1$ so $\frac{\Delta y}{\Delta x} = -\frac{1}{6}$

2) Use Form $Ax + By = C$:Think of it as Δy(x) + Δx(y) = C

$1x + 6y = C$: When the slope is **negative**, standard form has **(+)** in front of the y - term

$1(4) + 6(2) = C$: Plug in either set of (x , y) values

$4 + 12 = C$: Solve for C

$16 = C$

$1x + 6y = 16$:Write the **Standard** Form of equation.

VI Standard Equation From A Point and A Slope

A. Could write the equation in slope intercept form first then convert to standard
(Process already described in these notes)

B. Use knowledge of Standard Form to directly write equation (optional method)

Ex. 1 Given m = $\frac{3}{5}$ and (2 , 7),

Use Form	$Ax + By = C$	:Think of it as Δy(x) + Δx(y) = C
	$3x - 5y = C$	: When the slope is **positive**, standard form has **(–)** in front of the y - term
	$3(2) - 5(7) = C$	: Plug in either set of (x , y) values
	$6 - 35 = C$	: Solve for C
	$-29 = C$	
	$3x - 5y = -29$	:Write the **Standard** Form of equation.

Ex. 2 Given m = $-\frac{5}{2}$ and (3 , 1),

Use Form	$Ax + By = C$	:Think of it as Δy(x) + Δx(y) = C
	$5x + 2y = C$	: When the slope is **negative**, standard form has **(+)** in front of the y - term
	$5(3) + 2(1) = C$	: Plug in either set of (x , y) values
	$15 + 2 = C$	: Solve for C
	$17 = C$	
	$5x + 2y = 17$	:Write the **Standard** Form of equation.

C. Point Slope Form of Linear Equations

I. Forms of Equations

A. $(y-k)=m(x-h)$: Slope = $m=\frac{\Delta y}{\Delta x}=\frac{rise}{run}$

B. $(y-k)=\frac{\Delta y}{\Delta x}(x-h)$: Point (h , k)

II. Writing Equations

Ex. 1 Given m = $\frac{3}{4}$ and (2 , 7),

Use Form $(y-k)=\frac{\Delta y}{\Delta x}(x-h)$: Substitute the slope directly into the formula

$(y-7)=\frac{3}{4}(x-2)$: Plug in the set of (x , y) values where **h is the opposite of x** and **k is the opposite of y.**

Ex. 2 Given m = $-\frac{4}{5}$ and (5 , –3),

Use Form $(y-k)=\frac{\Delta y}{\Delta x}(x-h)$: Substitute the slope directly into the formula

$(y+3)=-\frac{4}{5}(x-5)$: Plug in the set of (x , y) values where **h is the opposite of x** and **k is the opposite of y.**

Ex. 2 Given m = $-\frac{2}{3}$ and (–2 , 4),

Use Form $(y-k)=\frac{\Delta y}{\Delta x}(x-h)$: Substitute the slope directly into the formula

$(y-4)=-\frac{2}{3}(x+2)$: Plug in the set of (x , y) values where **h is the opposite of x** and **k is the opposite of y**