Notes on Solving Equations

General Guidelines for Solving Equations.

1. K.I.S.S. every problem - "Keep It Simple Silly." Always write the problem in simplest form. Avoid working with double signs, and difficult forms of numbers such as fractions or decimals.
2. The Goal of Algebra - When solving equations, the aim is to get the variable by itself. Each and every step should get one closer to accomplishing the goal of getting the variable by itself. In other words, each line in the equation solving process should be in a simpler form than the previous line.
3. Types of acceptable answers - Using the methods discussed in these notes one should be capable of finding solutions that are in the form of integers, fractions, or mixed numbers. At **no time** should answers be written as decimals or improper fractions.
4. Maintain a state of equilibrium – What is done to one side of the equation **must** be done to the other side**.**
5. Know your vocabulary –
    a. *Coefficient* – A number in front of a variable
    b. *Constant* – A number not associated with a variable.
    c. *Unique Solutions* – When most equations are solved the end result is a single value. The significance of that value is that it is the only value that makes the equation, when viewed as an "open sentence", true. Examples: $y = 2\frac{3}{4}$, $x = -5$, or $w = -\frac{2}{3}$.
    d. *Infinite Solutions* – When solving equations on occasion the variables will drop out of the equation. Usually there is a step in the process where the "reflexive property" reveals itself prior to the variables dropping out. The remaining numbers produce a **true statement**. This result is identified as having an infinite number of solutions. Any number from the "set of real numbers" will make the equation, when viewed as an "open sentence", true. Use the symbol $\mathbb{R}$ to identify this type of result.
    e. *The Empty Set* – When solving equations on occasion the variables will drop out of the equation. When the remaining numbers produce a **false statement** it signifies that no number can ever solve the equation. When viewing this result in the context of "open sentences", it signals that there are no values that can ever make the original equation true. Use the symbol { } to identify this type of result.

Guided Examples For Solving Equations.

Example 1

$K - (-4) = 24$	: K.I.S.S. , write in simplest form
$K + 4 = 24$	:
$K = 24 - 4$	: Subtract 4 from each side
$K = 20$	: Simplify

Example 2

$34 = 15 - p$	: Subtract 15 from both sides
$34 - 15 = -p$	: Remember to always write original term first, migrating term second.
$19 = -p$	: Always solve for positive variables.
$-19 = p$	

Example 3

$v \div \frac{2}{3} = -\frac{4}{9}$	: K.I.S.S. – there is no dividing fractions
$\frac{3}{2}v = -\frac{4}{9}$	: Write the reciprocal as a coefficient
$18 \bullet \left[\frac{3}{2}v = -\frac{4}{9}\right]$	: To remove the fractions from the problem multiply by the common denominator. (Easier to divide then multiply)
$27v = -8$	
$\frac{27v}{27} = \frac{-8}{27}$	: Divide by the coefficient
$v = \frac{-8}{27}$	

Example 4

$5z + 9 = 20$	: In two-step equations, move constant first by,
$5z = 20 - 9$	: Subtracting 9 from each side, write the original term first and migrating term second.
$5z = 11$	
$\frac{5z}{5} = \frac{11}{5}$	: Divide by the coefficient
$z = 2\frac{1}{5}$	: Answer as mixed number

Example 5

$4r + (-3) = -1$	: K.I.S.S., write in simplest form
$4r - 3 = -1$	
$4r = -1 + 3$	: Add 3 to both sides of equation, write the original term first and the migrating term second.
$4r = 2$	
$\frac{4r}{4} = \frac{2}{4}$	: Divide by the coefficient
$r = \frac{1}{2}$	: Write answer as reduced fraction

Example 6

$8 - 3f = 23$

$-3f = 23 - 8$

$-3f = 15$

$\frac{-3f}{-3} = \frac{15}{-3}$

$f = -5$

: Subtract 8 from each side of equation, write the original term first and the migrating term second.
: Be sure to not lose the negative on the coefficient!

Example 7

$2.1 + .2g = 6.5$

$10[2.1 + .2g = 6.5]$

$21 + 2g = 65$

$2g = 65 - 21$

$2g = 44$

$\frac{2g}{2} = \frac{44}{2}$

$g = 22$

: To remove decimals from equation multiply by appropriate power of ten
:
: Subtract the constant from each side of the equation, write the original term first and the migrating term second.
:
: Divide by the coefficient.

Example 8

$\frac{y}{5} + 7 = -4$

$\frac{y}{5} = -4 - 7$

$\frac{y}{5} = -11$

$5 \bullet \left[\frac{y}{5} = -11\right]$

$y = -55$

: Move the constant first since it can easily be subtracted from both sides of the equation.
: Again, write the original term first and the migrating term second.
:
:
: To remove the fraction, multiply the equation by the common denominator.

Example 9

$11 = 9 - \frac{w}{3}$

$3 \cdot \left[11 = 9 - \frac{w}{3}\right]$

$33 = 27 - w$

$33 - 27 = -w$

$6 = -w$

$-6 = w$

: As an alternate approach to the previous example, multiply the equation by the common denominator.
: Be sure to distribute to each and every term.
:
: Make sure the variable does not lose the negative sign.
: Be sure to solve for positive variable in the end.

Example 10

$$\frac{3t+14}{5}=7$$

: The first step must be to multiply the equation by the common denominator which in this case is 5.

$$5\cdot\left[\frac{3t+14}{5}=7\right]$$

:

$$3t+14=35$$

:

: The numerator must come down unchanged in these types of problems.

$$3t=35-14$$

: Move the constant over to the other side by subtracting 14 to each side of the equation.

$$3t=21$$

$$\frac{3t}{3}=\frac{21}{3}$$

: Divide by the coefficient and get a final result.

$$t=7$$

Example 11

$$\frac{5-4f}{-3}=-9$$

: The first step must be to multiply the equation by the common denominator which in this case is –3.

$$-3\cdot\left[\frac{5-4f}{-3}=-9\right]$$

:

$$5-4f=27$$

:

: Move the constant over to the other side by subtracting 5 to each side of the equation. Again, write the original term first and the migrating term second.

$$-4f=27-5$$

$$-4f=22$$

: Be sure to not lose the negative on the coefficient!

$$\frac{-4f}{-4}=\frac{22}{-4}$$

: Divide, reduce and write answer as a mixed number.

$$f=-5\frac{1}{2}$$

Example 12

$$\frac{3}{7}(21m+14)-7m=32$$

: Notice that the terms inside the parentheses are multiples of seven, with such an occurrence it is better to distribute the fraction.

$$9m+6-7m=32$$

: Be sure to combine like terms

$$2m+6=32$$

: Subtract six from each side of the equation.

$$2m=32-6$$

$$2m=26$$

:

$$\frac{2m}{2}=\frac{26}{2}$$

: Divide by the coefficient.

$$m=13$$

Example 13

$5(2b+3)-10b=15$: Distribute the 5 then combine like terms.

$10b+15-10b=15$

$15=15$: When the variables drop out, evaluate the remaining statement. In this case the statement is true, so any real number will solve the equation. Use $\mathbb{R}$ to indicate such a result.

$\mathbb{R}$

Example 14

$-4g+3(2g-5)=31$: Distribute the 3 then combine like terms.

$-4g+6g-15=31$

$2g-15=31$: Add 15 to each side of equation. Write the original term first and the migrating term second.

$2g=31+15$:

$2g=46$:

$\frac{2g}{2}=\frac{46}{2}$: Divide by 2 and write final result.

$g=23$

Example 15

$\frac{3}{5}(5h-40)+8h=\frac{-2}{3}(6h-9)+5$: Distribute as the parentheses hold multiples of the denominators.

$3h-24+8h=-4h+6+5$: Combine like terms prior to moving any terms!

$11h-24=-4h+11$: Always move the smaller coefficient first, when attempting to get the variables to on one side of the equation. (Add 4h to both sides) Move any left over constants to the opposite side of the equation. (Add 24 to both sides)

$11h+4h=11+24$

$15h=35$: Divide, then write final answer as a reduced mixed number.

$\frac{15h}{15}=\frac{35}{15}$

$h=2\frac{1}{3}$

Example 16

$2(y-8)+7=5(y+2)-3y-19$: Distribute, then combine any like terms on each side of the equation.

$2y-16+7=5y+10-3y-19$: Notice this step is an example of the reflexive property.

$2y-9=2y-9$: Get the variables to one side of the equation and the constants to the other.

$2y-2y=-9+9$

$0=0$: Variables drop out and the remaining statement is true.

$\mathbb{R}$

Example 17

$3(z-2)+7=5(2z+3)-7z-10$: Distribute, then combine any like terms on each side of the equation.

$3z-6+7=10z+15-7z-10$:

$3z-1=3z+5$: Get the variables to one side of the equation and the constants to the other.

$3z-3z=5+1$: Variables drop out and the remaining statement is false. This implies that there are no numbers that can solve the original equation.

$0=6$

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Example 18

$\frac{1}{3}m+\frac{3}{4}=\frac{5}{6}m+\frac{1}{4}$: There are numerous fractions present, so multiply by the common denominator to simplify the equation.

$12\cdot\left[\frac{1}{3}m+\frac{3}{4}=\frac{5}{6}m+\frac{1}{4}\right]$:

$4m+9=10m+3$: Always move the smaller coefficient when attempting to get the variables to on one side of the equation. (Subtract 4m from both sides) Move any left over constants to the opposite side of the equation. (Subtract 3 from both sides)

$9-3=10m-4m$

$6=6m$: Divide by the coefficient

$\frac{6}{6}=\frac{6m}{6}$

$1=m$

Example 19

$\frac{3}{5}g-\frac{2}{3}=\frac{4}{5}g+2$: There are numerous fractions present, so multiply by the common denominator to simplify the equation. Be sure to multiply each term!

$15\cdot\left[\frac{3}{5}g-\frac{2}{3}=\frac{4}{5}g+2\right]$: Always move the smaller coefficient when attempting to get the variables to on one side of the equation. (Subtract 8g from both sides) Move any left over constants to the opposite side of the equation. (Add 10 from both sides)

$9g-10=8g+30$

$9g-8g=30+10$

$g=40$

Example 20

$$\frac{6b-7}{12}=\frac{5b+(-3)}{4}-\frac{4b-5}{3}$$

: The first step must be to multiply the equation by the common denominator which in this case is 12.

$$12\cdot\left[\frac{6b+11}{12}=\frac{5b+(-3)}{4}-\frac{4b-5}{3}\right]$$

: The numerator must come down unchanged in these types of problems, However there may be a remaining value that still needs to be distributed.

$$6b+11=3(5b-3)-4(4b-5)$$

$$6b+11=15b-9-16b+20$$

$$6b+11=-b+11$$

: Always move the smaller coefficient when attempting to get the variables to on one side of the equation. (Add b to both sides) Move any left over constants to the opposite side of the equation. (Subtract 11 from both sides)

$$6b+b=11-11$$

$$7b=0$$

$$\frac{7b}{7}=\frac{0}{7}$$

: **Zero** divided by any other number is **zero!**

$$b=0$$

Example 21

$$\frac{11r+2}{5}-\frac{3r-6}{4}=\frac{4r-3}{2}$$

: The first step must be to multiply the equation by the common denominator which in this case is 20.

$$20\cdot\left[\frac{11r+2}{5}-\frac{3r-6}{4}=\frac{4r-3}{2}\right]$$

: The numerator must come down unchanged in these types of problems, However there may be a remaining value that still needs to be distributed.

$$4(11r+2)-5(3r-6)=10(4r-3)$$

$$44r+8-15r+30=40r-30$$

$$29r+38=40r-30$$

: Always move the smaller coefficient when attempting to get the variables to on one side of the equation. Move any left over constants to the opposite side of the equation. Recall that the original terms should always be written first and the migrating terms second for each side of the equation.

$$38+30=40r-29r$$

$$68=11r$$

$$\frac{68}{11}=\frac{11r}{11}$$

: Divide and write answer as a mixed number.

$$6\frac{2}{11}=r$$