

## Factoring Using the Distributive Property (Undistributing)

<p>Ex 1    <math>8a^2 - 4a</math></p>  <p style="text-align: center;"><math>4a ( 2a - 1 )</math></p>	<p>: Find the GCF between terms. What is the largest number that can divide both 8 and 4, and what variable(s) do the terms have in common.</p> <p>: Pull “4a” out and ask, 4a times what equals <math>8a^2</math>? Then 4a times what equals <math>-4a</math>? Hopefully one arrives at 2a and <math>-1</math> respectively. Those results go into the parentheses.</p> <p>*To check answers one would only need to distribute final answer.</p>
<p>Ex 2    <math>6x^2y - 22xy</math></p>  <p style="text-align: center;"><math>2xy ( 3x - 11 )</math></p>	<p>: Find the GCF between terms. What is the largest number that can divide both 6 and 22, and what variable(s) do the terms have in common.</p> <p>: Pull “2xy” out and ask, 2xy times what equals <math>6x^2y</math>? Then 2xy times what equals <math>-22xy</math>? Hopefully one arrives at 3x and <math>-11</math> respectively. Those results go into the parentheses.</p> <p>*To check answers one would only need to distribute final answer.</p>
<p>Ex 3    <math>7wx^2y - 21xy^2</math></p>  <p style="text-align: center;"><math>7xy ( wx - 3y )</math></p>	<p>: Find the GCF between terms. What is the largest number that can divide both 7 and 21, and what variable(s) do the terms have in common.</p> <p>: Notice that what is left in parentheses MUST have a GCF of 1, in other words, those remaining terms have nothing in common to be pulled out.</p>
<p>Ex 4    <math>5x^3 - 10x^2 + 20xy</math></p>  <p style="text-align: center;"><math>5x ( x^2 - 2x + 4y )</math></p>	<p>: Find the GCF between terms.</p> <p>: Write remaining portion of terms in parentheses while ensuring that what remains has a GCF of 1.</p>
<p>Ex 5    <math>11x + 44xy</math></p>  <p style="text-align: center;"><math>11x ( 1 + 4y )</math></p>	<p>: Find the GCF between terms.</p> <p>: Write remaining portion of terms in parentheses while ensuring that what remains has a GCF of 1.</p> <p>*Please note that in this example the 1 inside the parentheses was needed as a place holder. This was very similar to example 2.</p>

## Factoring by Grouping

<p>Ex 1 <math>10AB + 15A - 6b - 9</math></p> <p><math>[ 10AB + 15A ] [ - 6b - 9 ]</math></p> <p><math>5A ( 2A + 3 ) - 3 ( 2A + 3 )</math></p> <p><math>( 5A - 3 )(2A + 3)</math></p>	<p>:First one should focus on two terms at a time so as to factor using the distributive property.</p> <p>:Notice that each pair of terms encloses the signs of the respective terms. THESE BRACKETS DO NOT IMPLY MULTIPLICATION but are used for grouping purposes.</p> <p>:From the first pair pull out 5A leaving <math>( 2B + 3)</math>. The goal is to match this set of parentheses when pulling something out of the second pair. In this case one should pull out a “-3”.</p> <p>:At this point bring the leading terms together in their own set of parentheses and rewrite what they have in common once.</p> <p>*These parentheses do imply multiplication. Checking the results can be done by double distribution or F.O.I.L.</p>
<p>Ex 2 <math>XY - 6Y - 2X + 12</math></p> <p><math>[ XY - 6Y ] [ - 2X + 12 ]</math></p> <p><math>Y (X - 6) - 2 (X - 6)</math></p> <p><math>( Y - 2 )(X - 6)</math></p>	<p>:Group the first two terms, then the last two terms.</p> <p>:Pull Y from the first pair</p> <p>:Pull -2 from the second pair to match <math>(X - 6)</math></p> <p>:At this point bring the leading terms together in their own set of parentheses and rewrite what they have in common once.</p> <p>*These parentheses do imply multiplication. Checking the results can be done by double distribution or F.O.I.L.</p>
<p>Ex 3 <math>W^2 - 5WZ + 3WZ - 15Z^2</math></p> <p><math>[ W^2 - 5WZ ] [ +3WZ - 15Z^2 ]</math></p> <p><math>W (W - 5Z) + 3Z (W - 5Z)</math></p> <p><math>( W + 3Z )(W - 5Z)</math></p>	<p>:Group the first two terms, then the last two terms.</p> <p>:Pull W from the first pair</p> <p>:Pull 3Z from the second pair to match <math>(W - 5Z)</math></p> <p>:At this point bring the leading terms together in their own set of parentheses and rewrite what they have in common once.</p> <p>*These parentheses do imply multiplication. Checking the results can be done by double distribution or F.O.I.L.</p>

<p>Ex 4 <math>2XR - 21 - 14R + 3X</math></p> <p><math>2XR - 14R + 3X - 21</math></p> <p><math>[2XR - 14R] [+3X - 21]</math></p> <p><math>2R(X - 7) + 3(X - 7)</math></p> <p><math>(2R + 3)(X - 7)</math></p>	<p>:Every once in a while the terms will be ordered in such a way that the first and last pair don't have much in common. Just rearrange the terms to make better pairs.</p> <p>:Group the first two terms, then the last two terms.</p> <p>:Pull 2R from the first pair</p> <p>:Pull 3 from the second pair to match <math>(X - 7)</math></p> <p>:At this point bring the leading terms together in their own set of parentheses and rewrite what they have in common once.</p> <p>*These parentheses do imply multiplication. Checking the results can be done by double distribution or F.O.I.L.</p>
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## Factoring Using AC Method (UnFOILing)

All Trinomials will take the form of  $Ax^2 + Bx + C$ , where A,B, and C are numerical values. This method requires the A and C values to be multiplied. The AC product is then broken down into pairs of factors that combine to produce the B coefficient.

<p>Ex 1 <math>x^2 + 8x + 15</math></p> <p><math>x^2 + 3x + 5x + 15</math></p> <p><math>[x^2 + 3x] [+ 5x + 15]</math></p> <p><math>x(x + 3) + 5(x + 3)</math></p> <p><math>(x + 5)(x + 3)</math></p>	<p>AC = 15 B = 8</p> <p>1, 15</p> <p><b>3, 5</b></p>	<p>The A coefficient is 1 and the C coefficient is 15. When multiplied it produces an AC value of 15. The AC value is <b>positive so both factors must have the same sign</b> as the middle term. Once identifying these factors use the values to rewrite the middle term. These are “the outer with outer” and “inner with inner” parts of F.O.I.L.</p> <p>After splitting the trinomial into four terms, follow the procedure for “factoring by grouping”.</p>
<p>Ex 2 <math>x^2 + 4x - 45</math></p> <p><math>x^2 - 5x + 9x - 45</math></p> <p><math>[x^2 - 5x] [+ 9x - 45]</math></p> <p><math>x(x - 5) + 9(x - 5)</math></p> <p><math>(x + 9)(x - 5)</math></p>	<p>AC = -45 B = 4</p> <p>-1, 45</p> <p>-3, 15</p> <p><b>-5, 9</b></p>	<p>The A coefficient is 1 and the C coefficient is -45. When multiplied it produces an AC value of -45. The AC value is <b>negative so two signs will be needed. The larger factor will match the sign of the middle term.</b> Once identifying these factors use the values to rewrite the middle term. These are “the outer with outer” and “inner with inner” parts of F.O.I.L.</p> <p>After splitting the trinomial into four terms, follow the procedure for “factoring by grouping”.</p>
<p>Ex 3 <math>x^2 - 10x + 21</math></p> <p><math>x^2 - 3x - 7x + 21</math></p> <p><math>[x^2 - 3x] [- 7x + 21]</math></p> <p><math>x(x - 3) - 7(x - 3)</math></p> <p><math>(x - 7)(x - 3)</math></p>	<p>AC = 21 B = -10</p> <p>-1, -21</p> <p><b>-3, -7</b></p>	<p>The A coefficient is 1 and the C coefficient is 21. When multiplied it produces an AC value of 21. The AC value is positive so both factors must have the same sign as the middle term. Once identifying these factors use the values to rewrite the middle term. These are “the outer with outer” and “inner with inner” parts of F.O.I.L.</p> <p>After splitting the trinomial into four terms, follow the procedure for “factoring by grouping”.</p>

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<p>Ex 4 <math>4x^2 + 20x + 21</math></p> <p><math>4x^2 + 6x + 14x + 21</math></p> <p><math>[4x^2 + 6x] [+14x + 21]</math></p> <p><math>2x(2x + 3) + 7(2x + 3)</math></p> <p><math>(2x + 7)(2x + 3)</math></p>	<p>AC = 84 B = 20</p> <p>1, 84</p> <p>2, 42</p> <p>3, 28</p> <p>4, 21</p> <p><b>6, 14</b></p>	<p>The A coefficient is 1 and the C coefficient is 15. When multiplied it produces an AC value of 15. The AC value is <b>positive so both factors must have the same sign</b> as the middle term. Once identifying these factors use the values to rewrite the middle term. These are “the outer with outer” and “inner with inner” parts of F.O.I.L.</p> <p>After splitting the trinomial into four terms, follow the procedure for “factoring by grouping”.</p>
<p>Ex 5 <math>6x^2 - 11x - 10</math></p> <p><math>6x^2 - 15x + 4x - 10</math></p> <p><math>[6x^2 - 15x] [+4x - 10]</math></p> <p><math>3x(2x - 5) + 2(2x - 5)</math></p> <p><math>(3x + 2)(2x - 5)</math></p>	<p>AC = -60 B = -11</p> <p>1, -60</p> <p>2, -30</p> <p>3, -20</p> <p><b>4, -15</b></p>	<p>The A coefficient is 1 and the C coefficient is -60. When multiplied it produces an AC value of -60. The AC value is <b>negative so two signs will be needed. The larger factor will match the sign of the middle term.</b> Once identifying these factors use the values to rewrite the middle term. These are “the outer with outer” and “inner with inner” parts of F.O.I.L.</p> <p>After splitting the trinomial into four terms, follow the procedure for “factoring by grouping”.</p>
<p>Ex 6 <math>12x^2 - 29x + 15</math></p> <p><math>12x^2 - 9x - 20x + 15</math></p> <p><math>[12x^2 - 9x] [-20x + 15]</math></p> <p><math>3x(4x - 3) - 5(4x - 3)</math></p> <p><math>(3x - 5)(4x - 3)</math></p>	<p>AC = 180 B = -29</p> <p>-1, -180</p> <p>-2, -90</p> <p>-3, -60</p> <p>-4, -90</p> <p>-5, -36</p> <p>-6, -30</p> <p><b>-9, -20</b></p>	<p>The A coefficient is 1 and the C coefficient is 21. When multiplied it produces an AC value of 21. The AC value is positive so both factors must have the same sign as the middle term. Once identifying these factors use the values to rewrite the middle term. These are “the outer with outer” and “inner with inner” parts of F.O.I.L.</p> <p>After splitting the trinomial into four terms, follow the procedure for “factoring by grouping”.</p>

## Factoring Using Difference of Two Squares (UnFOILing)

This is a special product that resulted in in two square products accompanied by one subtraction sign. Literally, this is a difference of two square values.

Ex 1 $25x^2 - 49$ $(5x + 7)(5x - 7)$	Take the square root of each term. $5x$ multiplied by itself produces $25x^2$ and $-7$ multiplied by $+7$ produced $-49$ . <b>Remember that the difference of two squares will produce two sets of parentheses and two different signs.</b>
Ex 2 $225x^2 - 169$ $(15x + 13)(15x - 13)$	Take the square root of each term. $15x$ multiplied by itself produces $225x^2$ and $-13$ multiplied by $+13$ produced $-169$ . Remember that the difference of two squares will produce two sets of parentheses and two different signs.
Ex 3 $64x^2 - 25y^2$ $(8x + 5y)(8x - 5y)$	Take the square root of each term. $8x$ multiplied by itself produces $64x^2$ and $-5y$ multiplied by $+5y$ produced $-25y^2$ . Remember that the difference of two squares will produce two sets of parentheses and two different signs.
Ex 4 $-121 + 81x^2$ $81x^2 - 121$ $(9x + 11)(9x - 11)$	Once in a while the terms may be switched in the problem. Remember that the only requirement is to have one subtraction sign and two square vales. In this case rewrite the terms so that the subtraction sign is between the terms and then follow the same steps from the previous examples.
Ex 5 $-256 + 529x^2$ $529x^2 - 256$ $(23x + 16)(23x - 16)$	Again the terms may be switched in the problem. Remember that the only requirement is to have one subtraction sign and two square vales. In this case rewrite the terms so that the subtraction sign is between the terms and then follow the same steps from the previous examples.  <i>*Notice that it has been requested that each student know the squares from <math>1^2</math> through <math>25^2</math> since the second week of school.</i>

## Factoring Using Perfect Squares (UnFOILing)

This is a special product that resulted in in two square products accompanied by a middle term that was exactly twice the product of the square roots of the first and last term. The title comes from the idea that the factored form will essentially have the same terms in the parentheses. In other words it is something multiplied by itself.

<p>Ex 1 <math>25x^2 - 70x + 49</math> <math>(5x - 7)(5x - 7)</math></p>	<p>Take the square root of the first and last term. <math>5x</math> multiplied by itself produces <math>25x^2</math> and <math>7</math> multiplied by itself produces <math>49</math>. <b>Notice that <math>5x</math> multiplied by <math>-7</math> produces <math>-35x</math>. When this is doubled (multiplied by two) the result is the middle term.</b> Write the same terms in two parentheses.</p>
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<p>Ex 2 <math>225x^2 + 390x + 169</math> <math>(15x + 13)(15x + 13)</math></p>	<p>The AC method would be a poor choice due the the nature of that product. When a trinomial produces an exceptionally large AC value it generally means that either there is a number that needs to be factored out or that the trinomial is a <b>perfect square</b>.</p> <p>Take the square root of the first and last term. <math>15x</math> multiplied by itself produces <math>225x^2</math> and <math>13</math> multiplied by itself produces <math>169</math>. Notice that <math>15x</math> multiplied by <math>13</math> produces <math>195x</math>. When this is doubled (multiplied by two) the result is the middle term of <math>390x</math>.</p>
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<p>Ex 3 <math>64x^2 - 80xy + 25y^2</math> <math>(8x - 5y)(8x - 5y)</math></p>	<p>Take the square root of each term. <math>8x</math> multiplied by itself produces <math>64x^2</math> and <math>5y</math> multiplied by itself produces <math>25y^2</math>. Notice that <math>8x</math> multiplied by <math>5y</math> produces <math>40xy</math>. When this is doubled (multiplied by two) the result is the middle term of <math>80xy</math>.</p> <p><b>Examine the results of these examples so far. The first and last terms are + squares and the middle term dictates the sign that is found in the resulting parentheses. So when the middle term is negative expect two negative signs in the end result. The same can be stated for positive middle terms.</b></p>
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<p>Ex 5 <math>9x^2 - 102x + 289</math> <math>(3x - 17)(3x - 17)</math></p>	<p>Take the square root of first and last term. Check to make sure that the product of those roots, when doubled, matches the middle term. Write the square roots in the parentheses being sure that the sign matches the middle term.</p>
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<p>Ex 5 <math>81x^2 + 198x + 121</math> <math>(9x + 11y)(9x + 11y)</math></p>	<p>Take the square root of first and last term. Check to make sure that the product of those roots, when doubled, matches the middle term. Write the square roots in the parentheses being sure that the sign matches the middle term.</p>
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